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16/ENG05/030
Mechatronics Engineering

Question 1

Define Mathematical Modeling

Outline two methods of obtaining models for engineering systems

Question 2

If

$r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t} K$

determine

i) $\frac{dr}{dt}$

ii) $\frac{d^2r}{dt^2}$ and

iii) the value of $\left|\frac{d^2r}{dt^2}\right|$ at $t = \tau$

Question 3

$A = x^2y i + (xy+yz) j + xz^2 K$

$B = yz i - 3xz j + 2xy K$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

determine at the point $(1,2,1)$

i) $\nabla\phi$

ii) $\nabla\cdot A$

(iii) $\nabla\times B$

(iv) grad div A,

(v) Curl Curl A

Mathematical Modelling:

Mathematical modelling can be defined as the process of using mathematical equation and interpreting the result f in physical or other terms (simulation)

Methods of Mathematical

a Forming a differential equation from an existing algebraic equation of the system.

b Using law of concentration of mass

Question 2

If $r = (t^2 + 3t)i - 2\sin 3t\, j + 3e^{2t} K$

$$\therefore \frac{dr}{dt} = (2t+3)i - (-6\cos 3t)j + 6e^{2t} K$$

$$\frac{dr}{dt} = (2t+3)i + (6\cos 3t)j + 6e^{2t} K$$

$$\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t} K$$

$$\frac{d^2r}{dt^2} \text{ at } t=0 = 2i + 18\sin 3(0) + 12e^{2(0)} K$$

$$= 2i + (18 \times 0)j + 12(1)K$$

$$= 2i + 12K$$

$$\therefore \left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2 + 12^2} = \sqrt{4+144} = \sqrt{148}$$

Question 3

$A = x^2y i + (xy + yz)j + xz^2 K$

$$\nabla \cdot A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial K}{\partial z}\right) x^2y i + (xy+yz)j + xz^2 K$$

$$= \left(\frac{\partial(x^2y)}{\partial x} + \frac{\partial(xy+yz)}{\partial y} + \frac{\partial \cdot (xz^2)}{\partial}\right)$$

$\nabla \cdot A = 2xy + x + z + 2xz$

at point $(1,2,1)$

$\nabla \cdot A = 2(1\times 2) + (1+1) + 2(1\times 1)$

$= 4 + 2 + 2$

$= \underline{8}$

$\nabla \cdot B = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot yzi - 3xzj + 2xyk$

$\nabla \cdot B = \left(\frac{\partial(yz)}{\partial x} + \frac{\partial(-3xz)}{\partial y} + \frac{\partial(2xy)}{\partial z}\right)$

$\nabla \cdot \phi = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot 3x^2y + xyz - 4y^2z^2 - 3$

$\nabla \cdot \phi = \left(\frac{\partial}{\partial x}(3x^2y + xyz - 4y^2z^2 - 3) + \frac{\partial}{\partial y}(3x^2y + xyz - 4y^2z^2 - 3) + \frac{\partial}{\partial z}(3x^2y + xy - 4y^2z^2 - 3)\right.$

$\nabla \cdot \phi = (6xy + yz)i + (3x^2 + xz - 8yz^2) + (xy - 8y^2z)$

at $(1,2,1)$

$= \left(6(1\times 2) + (2\times 1)\right)i + \left(3(1)^2 + (1\times 1) - 8(2\times 1^2)\right)j + \left((1\times 2) - 8(2)^2(1)\right)$

$= (12+2)i + (4 - 16)j + (2 - 32)k$

$= 14i + 12j - 30k$

$$\nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$$= i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ -3xz & 2xy \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ yz & 2xy \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ yz & -3xz \end{vmatrix}$$

$$i\left[\frac{\partial(2xy)}{\partial y} + \frac{\partial(3xz)}{\partial z}\right] - j\left[\frac{\partial(2xy)}{\partial x} - \frac{\partial(yz)}{\partial z}\right] + k\left[\frac{\partial(-3xz)}{\partial x} - \frac{\partial(yz)}{\partial y}\right]$$

$\nabla \times B = (2x+3x)i - (2y-y)j + (-3z-z)k$

$\nabla \times B = 5x\,i - yj - 4zk$

at $(1,2,1)$

$\nabla \times B = 5(1)i - 2j - 4(1)k$

$= 5i - 2j - 4k$

grad div A =

$div A = \nabla A$

$= \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot x^2yi + (xy+yz)j + xz^2k$

$= \left(\frac{\partial(x^2y)}{\partial x} + \frac{\partial}{\partial y}(xy+yz) + \frac{\partial}{\partial z}(xz^2)\right)$

$div A = 2xy + (x+z) + 2xz$

$\therefore$ grad div A $= \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot 2xy + (x+z) + 2xz$

$= \left(\frac{\partial}{\partial x}(2xy + (x+z) + 2xz)i + \frac{\partial}{\partial y}(2xy + (x+z) + 2xz)j + \frac{\partial}{\partial z}(2xy + x+z + 2xz)\right.$

$= (2y + 1 + 2z)i + (2x + \cancel{1 + 2x})j + (1 + 2x)k$

at $(1,2,1)$

$[2(2) + 1 + 2(1)]i + 2(1)j + (1 + 2(1))k$

$[4 + 1 + 2]i + 2j + 3k$

$7i + 2j + 3k$

Curl Curl A =

Curl Curl A

$$\text{Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & xy+yz & xz^2 \end{vmatrix}$$

$$\text{Curl } A = i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ xy+yz & xz^2 \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2y & xz^2 \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2y & xy+yz \end{vmatrix}$$

$$= \left[\frac{\partial(xz^2)}{\partial y} - \frac{\partial(xy+yz)}{\partial z}\right]i - \left[\frac{\partial(xz^2)}{\partial x} - \frac{\partial(x^2y)}{\partial z}\right]j$$

$$+ \left[\frac{\partial(xy+yz)}{\partial x} + \frac{\partial(x^2y)}{\partial y}\right]k$$

$$= (0 - y)i - (z^2 - 0)j + [y - x^2]k$$

$$= yi + z^2j + (y-x^2)k$$

$$\text{Curl Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & -z^2 & y-x^2 \end{vmatrix}$$

$$\text{Curl Curl } A = i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ -z^2 & y-x^2 \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ y & y-x^2 \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ y & -z^2 \end{vmatrix}$$

$$= \left[\frac{\partial(y-x^2)}{\partial y} - \frac{\partial(-z^2)}{\partial z}\right]i - \left(\frac{\partial(y-x^2)}{\partial x} - \frac{\partial(y)}{\partial z}\right)j$$

$$+ \left[\frac{\partial(-z^2)}{\partial x} - \frac{\partial(y)}{\partial y}\right]k$$

$$= [1 - 2z]i + [-2x - 1]j + (0-1)k$$

at $(1,2,1)$

$$= [1 - 2(1)]i - [-2(1) - 1]j + -1k$$

$$= [-1]i - [-3]j - k$$

$$= -i + 3j - k$$