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16/ENG03/020

CIVIL ENGINEERING

ENG 282

1. Mathematical modelling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers, and guidance useful for the originating application.

2. A model can be obtained by simulation

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Question 2.

$r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t} k$

i) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t} k$

ii) $\frac{d^2r}{dt^2} = (2)i + 18\sin 3t\, j + 12e^{2t} k$

iii) $\left|\frac{d^2r}{dt^2}\right|$ at $t=0$

$= 2i + 18\sin 3(0)\, j + 12e^{2(0)} k$

$= 2i + 0j + 12k$

$= \underline{\underline{2i + 12k}} = \sqrt{2^2+12^2}$

$= \sqrt{4+144}$

$= \sqrt{148}$

Question 3.

$A = x^2y\, i + (xy+yz)j + xz^2 k$

$B = yz\, i - 3xz\, j + 2xy\, k$ and

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$ at $\overset{x\ y\ z}{(1, 2, 1)}$

determine

i) $\nabla\phi = \left(\frac{\partial\phi}{\partial x}\right)i + \left(\frac{\partial\phi}{\partial y}\right)j + \left(\frac{\partial\phi}{\partial z}\right)k$

$= \left(\frac{\partial(3x^2y + xyz - 4y^2z^2 - 3)}{\partial x}\right)i + j\left(\frac{\partial(3x^2y + xyz - 4y^2z^2 - 3)}{\partial y}\right)$

$+ k\left(\frac{\partial(3x^2y + xyz - 4y^2z^2 - 3)}{\partial z}\right)$

$= (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)k$

$= (6(1\times2) + (2\times1))i + (3(1)^2 + (1\times1) - 8(2\times1^2))j + (1\times2 - 8\times2^2\times1)k$

$= \underline{\underline{14i - 12j - 30k}}$

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(ii) $\text{div } A = \nabla \cdot A$

$$= \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot \left(x^2 y i + (xy + yz) j + xz^2 k\right)$$

$$= \frac{\partial (x^2 y)}{\partial x} i + \frac{\partial (xy+yz)}{\partial y} j + \frac{\partial (xz^2)}{\partial z} K$$

$$= (2xy) + (x+z) + (2xz) \quad \text{at } (1,2,1)$$

$$= 2(1\times 2) + (1+1) + (2\times 1\times 1) = \underline{8}$$

(iii) $\nabla \times B = \text{Curl } B$

$$= \begin{vmatrix} i & j & K \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (yz) & (-3xz) & (2xy) \end{vmatrix}$$

$$= \left(\frac{\partial(2xy)}{\partial y} - \frac{\partial(-3xz)}{\partial z}\right) i - \left(\frac{\partial(2xy)}{\partial x} - \frac{\partial(yz)}{\partial z}\right) j$$

$$+ K \left(\frac{\partial(-3xz)}{\partial x} - \frac{\partial(yz)}{\partial y}\right)$$

$$= (2x + 3x) i - (2y - y) j + K(-3z - z)$$

$$= (5x) i - y j - (4z) K$$

$$= 5(1) i - (2) j - 4(1) K \quad \text{at } (1,2,1)$$

$$= \underline{\underline{5i - 2j - 4K}}$$

iv) $\text{Grad div } A = \nabla(\nabla \cdot A)$

$\nabla \cdot A = 2xy + x + z + 2xz$ — from (ii)

$$\therefore \text{Grad div } A = \frac{\partial (2xy + x + z + 2xz)}{\partial x} i + \frac{\partial (2xy + x + z + 2xz)}{\partial y} j$$

$$+ \frac{\partial (2xy + x + z + 2xz)}{\partial z} K$$

$$= (2y + 1 + 2z) i + (2x) j + (1 + 2x) K$$

$$= [2(2) + 1 + 2(1)] i + 2(1) j + (1 + 2(1)) K$$

$$= \underline{\underline{7i + 2j + 3K}}$$

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(v) Curl Curl A

$$A = (x^2y)i + (xy + yz)j + xz^2 K.$$

$$\text{Curl } A = \begin{vmatrix} i & j & K \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & xy+yz & xz^2 \end{vmatrix}$$

$$= i\left(\frac{\partial(xz^2)}{\partial y} - \frac{\partial(xy+yz)}{\partial z}\right) - j\left(\frac{\partial(xz^2)}{\partial x} - \frac{\partial(x^2y)}{\partial z}\right)$$

$$+ K\left(\frac{\partial(xy+yz)}{\partial x} - \frac{\partial(x^2y)}{\partial y}\right).$$

$$= i(0 - y) - j(z^2 - 0) + K(y - x^2)$$

$$\phi = -yi - z^2j + (y - x^2)K$$

Let Curl $A = \phi$

$$\nabla \times \phi = \begin{vmatrix} i & j & K \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & y-x^2 \end{vmatrix}$$

$$= i\left(\frac{\partial(y-x^2)}{\partial y} - \frac{\partial(-z^2)}{\partial z}\right) - j\left(\frac{\partial(y-x^2)}{\partial x} - \frac{\partial(-y)}{\partial z}\right)$$

$$+ K\left(\frac{\partial(-z^2)}{\partial x} - \frac{\partial(-y)}{\partial y}\right)$$

$$= i(1-(-2z)) - j(-2x) + K(1)$$

$$= i(1+2z) - j(-2x) + K.$$

at $(1, 2, 1)$

$$= i(1+2(1)) - j(-2(1)) + K$$

$$= i3 + 2j + K$$

$$= 3i + 2j + K.$$