1 a) A mathematical model is a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting results in physical and other terms.

b (i) Exponential growth / decay (use of ODE)
(ii) Mixing problems.

2) $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t}k$

i) $\frac{dr}{dt} = (2t+3)\hat{i} - 6\cos 3t\hat{j} + 6e^{2t}\hat{k}$

ii) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t\hat{j} + 12e^{2t}\hat{k}$

iii) $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2\hat{i} + 12\hat{k}$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2+12^2} = \sqrt{4+144}$

$= \sqrt{148}$

$= 2\sqrt{37}$

$= 12.17$

3) $A = x^2yi + (xy + yz)j + xz^2k$

$B = yzi - 3xzj + 2xyk$

$\emptyset = 3x^2y + xyz - 4y^2z^2 - 3$

i) $\nabla = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} \hat{k}$

$\frac{\partial \phi}{\partial x} = 6xy + yz$

$\frac{d\phi}{dz} = xy - 8y^2 z$

$\frac{d\phi}{dy} = 3x^2 + xz - 8yz^2$

At $(1, 2, 1)$

$\frac{d\phi}{\partial x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$

$\frac{d\phi}{\partial y} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3 + 1 - 16 = -12$

$\frac{d\phi}{dz} = (1)(2) - 8(2) + (1) = 2 - 32 = -30$

$\nabla \phi = 14i - 12j - 30\hat{k}$

ii) $\nabla \cdot A = \frac{\partial a}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$A = x i + a y j + z \hat{k}$

$\nabla \cdot A = 2xy + (x + 2) + 2xz$

At $(1, 1, 1)$

$\nabla \cdot A = \cancel{x2}\ 2(1)(2) + (1+1) + 2(1)(1)$

$= 4 + 2 + 2 = 8$

iii) $\nabla \times 6$ $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & \partial/\partial y & \partial/\partial z \\ yz^2 & -3xz & 2xy \end{vmatrix}$

$= \hat{i}(2x + 3x) - \hat{j}(2y - y) + \hat{k}(-3z - 2)$

$= 5x\hat{i} - 3y\hat{j} - 4z\hat{k}$

At $(1, 2, 1)$

$\nabla \times B = 5\hat{i} - 2\hat{j} - 4\hat{k}$

iv) grad div

grad $(2xy + (x+z) + 2xz)$

let div $A = C = \nabla A$

$$\nabla(\nabla A) = \nabla C = \hat{i}\frac{dc}{\partial x} + \hat{j}\frac{dc}{\partial y} + \hat{k}\frac{dc}{dz}$$

$$= \hat{i}(2y + 1 + 2z) + \hat{j}(2x) + \hat{k}(1 + 2x)$$

At $(1, 2, 1)$

$$\nabla C = \hat{i}(2(2) + 1 + 2(1)) + \hat{j}(2(1)) + \hat{k}(1 + (2)(1))$$

$$= \hat{i}(4 + 1 + 2) + \hat{j}(2) + \hat{k}(1+2)$$

$$= \cancel{7}\ 7\hat{i} + 2\hat{j} + 3\hat{k}$$

v)

$$A = \nabla \times A$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy + yz) & xz^2 \end{vmatrix}$$

$$= \hat{i}(0 - y) - \hat{j}(z^2 - 0) + \hat{k}(y - x^2)$$

$$= -y\hat{i} - z^2\hat{j} + \hat{k}(y - x^2)$$

At $(1, 2, 1)$

$$\text{Curl } A = -2\hat{i} - \hat{j} + \hat{k}$$

$$A = \nabla \times (\nabla \times A)$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y - x^2) \end{vmatrix}$$

$$= \hat{i}(1 + 2z) - \hat{j}(-2x - 0) + \hat{k}(0 + 1)$$

$$= \hat{i}(1 + 2z) + 2x\hat{j} + \hat{k}$$

$=$

At point (1, 2, 1)

$\nabla \times (\nabla \times A) = \hat{i}(1+2(1)) + 2(1)^2\hat{j} + \hat{k}$

$= 3\hat{i} + 2\hat{j} + \hat{k}$