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16/ENG06/057

Mechanical Engr.

Eng 282 .... Engr. Math (IV)

07/04/2018.

Assignment IV

1.) i.) Mathematical modelling:

The Process of developing a mathematical model is termed as "mathematical modelling". And a matematical model is a description of a System Using mathematical Concepts and Languages (ideas).

ii.) 2 methods of Obtaining models for Engr. Systems;

- Forming a differential equation from an existing algebraic equation of the System.
- Using the Balance Law.

2.) If $r = (t^2+3t)\hat{i} - 2\sin 3t\hat{j} + 3e^{2t}k$,

i) $\frac{dr}{dt} \Rightarrow (2t+3)\hat{i} - 6\cos 3t\hat{j} + 6e^{2t}k$.

ii) $\frac{d^2r}{dt^2} = 2\hat{i} + 18\sin 3t\hat{j} + 12e^{2t}k$.

iii) $\left|\frac{d^2r}{dt^2}\right|$ at $t=0$.

$\Rightarrow 2\hat{i} + 18\sin 3(0)\hat{j} + 12e^{2(0)}k$,

$\sqrt{(2)^2+(0)^2+(12)^2} \Rightarrow \sqrt{148}$

$\Rightarrow 2\sqrt{37}$.

3.) If $A = x^2y\hat{i} + (xy+yz)\hat{j} + xz^2\hat{k}$,

$B = \dots 3xz\hat{j} + 2xy\hat{k}$, and

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If $A = x^2y\hat{i} + (xy + yz)\hat{j} + xz^2\hat{k}$,

$B = yz\hat{i} - 3xz\hat{j} + 2xy\hat{k}$, and.

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$,

at Point $(1, 2, 1)$

$\begin{matrix} x & y & z \\ (1, & 2, & 1) \end{matrix}$

i) $\nabla\phi = \text{Grad}\,\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$

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$\frac{d\phi}{dx} \Rightarrow 6xy + yz\,\hat{i}$, $d\phi/dz \Rightarrow xy - 8y^2z$ $\quad \overset{x\,y\,z}{(1,2,1)}$

$\frac{d\phi}{dy} \Rightarrow 3x^2 + xz - 8yz^2$

$\nabla\phi = (6xy + yz)\hat{i} + (3x^2 + xz - 8yz^2)\hat{j} + (xy - 8y^2z)\hat{k}$.

$= [6(1)(2) + 2]\hat{i} + [3 + 1 - 8(2)(1)^2]\hat{j} + (1(2) - 8(2)^2(1))\hat{k}$.

$\nabla\phi = 14\hat{i} - 12\hat{j} - 30\hat{k}$.

$= 14\hat{i} - 12\hat{j} - 30\hat{k} \rightarrow (14, -12, -30)$.

ii) $\nabla \cdot A$

$\nabla = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left[x^2y\,\hat{i} + (xy + yz)\hat{j} + xz^2\hat{k}\right]$

$\begin{cases} i \cdot i = j \cdot j = \hat{k}\cdot\hat{k} = 1 \\ i\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{i}\cdot\hat{k} = 0 \end{cases}$

$\nabla \cdot A = \frac{\partial}{\partial x}(x^2y)\hat{i}\cdot\hat{i} + \frac{\partial}{\partial y}(xy + yz)\hat{j}\cdot\hat{j} + \frac{\partial}{\partial z}(xz^2)\hat{k}\cdot\hat{k}$.

$= 2xy + (x + z) + (2xz)$

at $(1, 2, 1)$
$\quad\ \ x\ y\ z$

$\nabla \cdot A = 2(1)(2) + (1 + 1) + (2(1)(1)) \rightarrow 8$

$(1, 2, 1)$
x, y, z

iii) $\nabla \times B$ $\quad + \ - \ +$

$\nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$

$= \hat{i}\left[2xy\,\partial/\partial y + 3xz\,\partial/\partial z\right] - \hat{j}\left[\partial/\partial x\, 2xy - yz\,\partial/\partial z\right] + \hat{k}\left[-3xz\,\partial/\partial x - yz\,\partial/\partial y\right]$

$= \hat{i}[2x + 3x] - \hat{j}[2y - y] + \hat{k}[-3z - z]$.

$= \hat{i}[2(1) + 3(1)] - \hat{j}[2(2) - 2] + \hat{k}[-3(1) - (1)]$.

$= \hat{i}[5] - \hat{j}[2] + \hat{k}[-4]$.

$\nabla \times B = 5\hat{i} - 2\hat{j} - 4\hat{k}$. $\Rightarrow [5, -2, -4]$. $\quad \overset{x\,y\,z}{(1,2,1)}$

iv) grad div A $\quad$ Let $\nabla \cdot A = Q = 2xy + (x + z) + (2xz)$

Grad div A $\Rightarrow \nabla(\nabla \cdot A) \rightarrow$

$\nabla Q = \frac{\partial Q}{\partial x}\hat{i} + \frac{\partial Q}{\partial y}\hat{j} + \frac{\partial Q}{\partial z}\hat{j}$

$\frac{\partial Q}{\partial x} = 2y + 1 + 2z \qquad \nabla Q = (2y + 2z + 1)\hat{i} + (2x)\hat{j} + (1 + 2x)\hat{k}$.

$\frac{\partial Q}{\partial y} = 2x \qquad = [2(2) + 1 + 2(1)]\hat{i} + [2]\hat{j} + [1 + 2(1)]\hat{k}$.

$\frac{\partial Q}{\partial z} = 1 + 2x \qquad = 7\hat{i} + 2\hat{j} + 3\hat{k} \cdot [7, 2, 3]$.

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v.) Curl Curl A.

$A = x^2y\hat{i} + (xy + yz)\hat{j} + xz^2\hat{k}$ $\quad (1, 2, 1)$

$$\text{Curl } A = \nabla \times A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix} \rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= \hat{i}\left[\partial/\partial y\,(xz^2) - \partial/\partial z\,[xy + yz]\right] - \hat{j}\left[(xz^2)\partial/\partial x - \partial/\partial z\,[x^2y]\right]$$

$$+ \hat{k}\left[\partial/\partial x\,(xy+yz) - \partial/\partial y\,(x^2y)\right].$$

$$= \hat{i}[-y] - \hat{j}[z^2] + \hat{k}[y - x^2].$$

$$= -y\hat{i} - z^2\hat{j} + (y - x^2)\hat{k}.$$

$$= -2\hat{i} - (1)^2\hat{j} + (2 - (1)^2)\hat{k}.$$

$$= -2\hat{i} - \hat{j} + \hat{k} \Rightarrow [-2, -1, 1].$$

$$\text{Curl Curl } A \rightarrow \nabla(\nabla \times A)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & (-z^2) & +(y-x^2) \end{vmatrix}$$

$$= \hat{i}\left[\partial/\partial y\,(y - x^2) + \partial/\partial z\,(+z^2)\right] - \hat{j}\left[\partial/\partial x\,(y - x^2) + \partial/\partial z\,(y)\right]$$

$$+ \hat{k}\left[\partial/\partial x\,(-z^2) + \partial/\partial y\,(y)\right].$$

$$= \hat{i}\left[[1] + 2z\right] - \hat{j}\left[[2x] + 0\right] + \hat{k}[1].$$

$$= (1 + 2z)\hat{i} - (2x)\hat{j} + \hat{k}.$$

$$= (1 + 2)\hat{i} - 2\hat{j} + \hat{k}.$$

$$= 3\hat{i} - 2\hat{j} + \hat{k} \Rightarrow (3, -2, 1).$$

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