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DEPT: Computer Engr
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Assignment

1a) A mathematical model is a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical and other terms.

b) i) Exponential Growth Decay (use of ODE)
ii) Mixing Problems

2 $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t}k$

i) $\frac{dr}{dt} = (2t + 3)i - 6\cos 3t j + 6e^{2t}k$

ii) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t}k$

iii) $\frac{d^2r}{dt^2}\Big|_{t=0} = 2i + 12k$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2+12^2} = \sqrt{4+144}$
$= \sqrt{148} = 2\sqrt{37} = 12.17$

3) $A = x^2y\,i + (xy + y^2)j + xz^2k$
$B = yz\,i - 3xz\,j + 2xy\,k$
$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

i) $\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$

$\frac{\partial\phi}{\partial x} = 6xy + yz$

$\frac{\partial \phi}{\partial z} = xy - 8y^2z$

$\frac{\partial \phi}{\partial y} = 3x^2 + xz - 8yz^2$

At $(1, 2, 1)$

$\frac{\partial \phi}{\partial x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$

$\frac{\partial \phi}{\partial y} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3 + 1 - 16$
$= -12$

$\frac{\partial \phi}{\partial z} = (1)(2) - 8(2)^2(1) = 2 - 32$
$= -30$

$\nabla \phi = 14i - 12j - 30k$

ii) $\nabla \cdot A = \frac{\partial a}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$A = a_x i + a_y j + a_z k$

$\nabla \cdot A = 2xy + (x + z) + 2xz$

At $(1, 2, 1)$

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$= 4 + 2 + 2 = 8$

iii) $\nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & 3xz & 2xy \end{vmatrix}$

$= i(2x + 3x) - j(2y - y) + k(-3z - z)$

$= 5xi - yj - 4zk$

At $(1, 2, 1)$

$\nabla \times B = 5i - 2j - 4k$

iv) grad div A

grad $(2xy + (x + z) + 2xz)$

let div A $= C = \nabla A$

$$\nabla(\nabla A) = \nabla c = i\frac{\partial c}{\partial x} + j\frac{\partial c}{\partial y} + k\frac{\partial c}{\partial z}$$

$$= i(2y+1+2z) + j(2x) + k(1+2x)$$

At $(1,2,1)$

$$\nabla c = i[2(2)+1+2(1)] + j[2(1)] + k[1+(2)(1)]$$

$$= i(4+1+2) + j(2) + k(1+2)$$

$$= 7i + 2j + 3k$$

v. Curl ~~Ca~~ A

$$\text{Curl } A = \nabla \times A$$

$$= \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dx} & \frac{d}{dz} \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= i(0-y) - j(z^2-0) + k(y-x^2)$$

$$= -yi - z^2j + k(y-x^2)$$

At $(1,2,1)$

$$\text{Curl } A = -2i - j + k$$

$$\text{Curl } A = \nabla \times (\nabla \times A)$$

$$\nabla \times (\nabla \times A) \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= i(1+2z) - j(-2x-0) + k(0+1)$$

$$= i(1+2z) + 2x^2j + k$$

At point $(1,2,1)$

$$\nabla \times (\nabla \times A) = i(1+2(1)) + 2(1)^2j + k$$

$$= 3i + 2j + k$$