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16/EN G04/1014
Elect

Q1

mathematical modeling can be defined as the process setting up a model, solving it mathematically and interpreting the result in physical or other terms

Q1b.

Two methods of obtaining models for engineering system are

- Using the balance law (law of conservation of mass)
- Forming a differential equation from an existing algebraic equation of the system

Q2

i) $r = (t^2+3t)i - (2\sin 3t)j + (3e^{2t})k$

recall that $\frac{dr}{dt} = i\frac{dx(t)}{dt} + j\frac{dy(t)}{dy} + k\frac{dz(t)}{dy}$

$\frac{dr}{dt} = (2t+3)i - (6\cos 3t)j + (6e^{2t})k$

ii) $\frac{d^2r}{dt^2} = (2)i + (18\cos 3t)j + (12e^{2t})k$

iii) $\left|\frac{d^2r}{dt^2}\right| = 0$

$t = 0$

$\frac{d^2r}{dt^2} = 2i + 0j + 12k$

$= 2i + 12k$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2+12^2}$

$=$

$= \sqrt{148}$

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$= i[0-y] - j[z^2-0] + k[y-x^2]$

$= -yi - z^2 j + y - x^2 k$

$\text{Curl } A = -yi - z^2 j + [y-x^2]k$

curl

$\text{Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & y-x^2 \end{vmatrix}$

$i\left[\frac{\partial}{\partial y}(y-x^2) - \frac{\partial}{\partial z}(-z^2)\right] - j\left[\frac{\partial}{\partial x}(y-x^2) - \frac{\partial}{\partial z}(-y)\right) + k\left[\frac{\partial}{\partial x}(-z^2 \quad \frac{\partial}{\partial y}\right.$

$= i[1+2z) - j(-2x] + k(1)$

$\text{Curl Curl } A = [1+2z)i + 2x + k$

at point 1, 2, 1

$= 3i + 2j + k$

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$\nabla \times B$

$= yz\,i - 3xz\,j + 2xy\,k$

$$B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$$i\left[\frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial z}(-3xz)\right] - j\left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(yz)\right] + k\left[\frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}(yz)\right]$$

$i[2x + 3x] - j\{2y - y\} + k[-3z - z]$

$= 5x\,i - 2j - 4k$ at point 1, 2, 1

$\times B = 5(1)i - 2j - 4k$

$= 5i - 2j - 4k$

) grad div A, $= \nabla\cdot(\nabla A) = \nabla^2 A$

grad div A, $\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z}$

$= x^2y\,i + (xy + yz)j + xz^2 k$

div $A = 2xy + (x + z) + 2zx$

grad div $A = i(2y + 1 + 2z) + (2x)j + (1 + 2x)k$

at point $(1, 2, 1)$

$= i(4 + 1 + 2) + (2)j + (1 + 2)k$

grad div $A = 7i + 2j + 3k$

) Curl Curl A

$$\text{Curl} A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy + yz) & xz^2 \end{vmatrix}$$

$$i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ xy + yz & xz^2 \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2y & xz^2 \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2y & xy + yz \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy + yz)\right] - j\left[\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right] + k\left[\frac{\partial}{\partial x}(xy + yz\right.$$

$$\left. - \frac{\partial}{\partial y}(x^2y)\right]$$

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$= \sqrt{148}$

$= 12.17$

Q3

$A = x^2y\,i + (xy + yz)\,j + xz^2\,k$

$B = 1z\,i - 3xz\,j + 2xy\,k$

$\phi = 3x^2y + xyz - 4y^2z^2 - 6$

at points $(1, 2, 1)$

i) $\nabla \phi$

$$\nabla \phi = \frac{\partial \phi i}{\partial x} + \frac{\partial \phi j}{\partial y} + \frac{\partial \phi k}{\partial k}$$

$= (6xy + yz - 0 - 0)i + (3x^2 + xz - 8yz^2 - 0)j +$ [illegible]

$(0 + xy - 8y^2z - 0)k$ at point $(1, 2, 1)$

$\nabla \phi = (6(1)(2) + (2)(1))i + (3(1)^2 + (1)(1) - 8(2)(1)^2)j +$ [illegible]

$= (12 + 2)i + (3 + 1 - 16)j + (2 - 32)k$

$\nabla \phi = 14i - 12j - 30k$

$= 2(7i - 6j - 15k)$

ii) ∇A

$$\nabla A = \frac{\partial ax}{\partial x} + \frac{\partial ay}{\partial y} + \frac{\partial az}{\partial z}$$

$A = x^2y\,i + (xy + yz)\,j + xz^2\,k$

$ax = x^2y$

$ay = (xy + yz)$

$az = (xz^2)$

$\nabla A = 2xy + (x + z) + 2xz$

at point $(1, 2, 1)$

$= 2(1)(2) + (1 + 1) + 2(1)(1)$

$= 4 + 2 + 2$

$= 8$

$\nabla A = 8$

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