

$$i \left[ \frac{\partial (2xy^2)}{\partial y} - \frac{\partial (-3xz)}{\partial z} \right] - j \left[ \frac{\partial (2xy^2)}{\partial x} - \frac{\partial (yz^2)}{\partial z} \right] + k \left[ \frac{\partial (-3xz)}{\partial x} - \frac{\partial (yz^2)}{\partial y} \right]$$

$$= (2x + 3x)i - j(2y - y) + k(-3z - z)$$

$\nabla \times B$  at  $(7, 2, 1)$

$$\nabla \times B = (2 \times 1 + 3 \times 1)i - j(2 \times 2 - 2) + k(-3 \times 1 - 1)$$

$$\nabla \times B = (5)i - j(2) + k(-4)$$

$$\nabla \times B = 5i - 2j - 4k$$

Grad of div A

$$\nabla \cdot A = 2xy + (5x + 2) + 2xz$$

$$\nabla A = \left[ i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right] \nabla \cdot A$$

$$\nabla A = \left[ (2y + 5 + 2z)i + j(2x) + k(1 + 2x) \right]$$

$\nabla(\nabla \cdot A)$  at  $(7, 2, 1)$

$$\nabla(\nabla \cdot A) = (2 \times 2 + 1 + 2 \times 1)i + (2 \times 1)j + (1 + 2 \times 1)k$$

$$= (7)i + (2)j + 3k$$

$$7i + 2j + 3k$$

Curl/Curl A

$$\text{Curl } A = \nabla \times A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & (xy + y^2) & (xz^2) \end{vmatrix}$$

$$i \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (xy + y^2) & (xz^2) \end{vmatrix} - j \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ x^2y & (xz^2) \end{vmatrix} + k \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x^2y & (xy + y^2) \end{vmatrix}$$

$$i \left[ \frac{\partial (xz^2)}{\partial y} - \frac{\partial (xy + y^2)}{\partial z} \right] - j \left[ \frac{\partial (xz^2)}{\partial x} - \frac{\partial (x^2y)}{\partial z} \right] + k \left[ \frac{\partial (xy + y^2)}{\partial x} - \frac{\partial (x^2y)}{\partial y} \right]$$

$$i [0 - y] - j [z^2 - 0] + k [y - x^2]$$

at  $(1, 2, 1)$

$$\vec{A} = -y\mathbf{i} - z^2\mathbf{j} + (y-x^2)\mathbf{k}$$

$$\nabla \times (\nabla \times \vec{A}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$\mathbf{i} \left| \frac{\partial}{\partial y} (y-x^2) - \frac{\partial}{\partial z} (-z^2) \right| - \mathbf{j} \left| \frac{\partial}{\partial x} (y-x^2) - \frac{\partial}{\partial z} (-y) \right|$$

$$+ \mathbf{k} \left| \frac{\partial}{\partial x} (-z^2) - \frac{\partial}{\partial y} (-y) \right|$$

1) Mathematical modelling is a description of a system using mathematical concepts and/or mathematical models as a process of developing a mathematical model.

Methods of developing model in engineering system

- 1) Feasibility
- 2) Mixing problems

Question 2

$$x = (t^2 + 3t) i - 3 \sin 3t j + 3e^{2t} k$$

$$a) \frac{dx}{dt} = (2t + 3) i - (3 \cos 3t) j + (6e^{2t}) k$$

$$b) \frac{d^2x}{dt^2} = (2) i + (18 \sin 3t) j + (12e^{2t}) k$$

$$\frac{d^2x}{dt^2} \text{ at } t=0 = 2i + 18 \sin 0 j + 12e^0 k$$

$$\frac{d^2x}{dt^2} \text{ at } t=0 = 2i + 12k$$

$$\left| \frac{d^2x}{dt^2} \right|_{t=0} = \sqrt{2^2 + 12^2}$$

$$\left| \frac{d^2x}{dt^2} \right|_{t=0} = \sqrt{4 + 144} = \sqrt{148} = 12.2 \text{ units}$$

$$\nabla\phi = \left[ i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right] \phi$$

$$\nabla\phi = i \frac{\partial\phi}{\partial x} + j \frac{\partial\phi}{\partial y} + k \frac{\partial\phi}{\partial z}$$

$$\nabla\phi = i \left[ yz + 8yz \right] + j \left[ 3x^2 + xz - 8y^2z \right] + k \left[ xy - 8y^2z \right]$$

at point (1, 2, 1)

$$\nabla\phi = i \left[ 2 \times 1 + 1 \times 2 \right] + j \left[ 3 \times 1^2 + 1 \times 1 - 8 \times 2 \times 1^2 \right] + k \left[ 1 \times 2 - 8 \times 2^2 \times 1 \right]$$

$$\nabla\phi = i(4) + j(-12) + k(-30)$$

$$\nabla\phi = 2i + 12j + 30k$$

$\nabla \cdot A$

$$\nabla = i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z}$$

$$A = axi + ayj + azk$$

$$\nabla \cdot A = \left[ i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right] \cdot [axi + ayj + azk]$$

$$\nabla \cdot A = \frac{\partial}{\partial x} \cdot ax + \frac{\partial}{\partial y} \cdot ay + \frac{\partial}{\partial z} \cdot az$$

$$= 2xy + (x+2) + 2xz$$

$$\nabla \cdot A \text{ at } (1, 2, 1)$$

$$\nabla \cdot A = 2 \times 1 \times 2 + (1+1) + 2 \times 1 \times 1$$

$$\nabla \cdot A = 4 + 2 + 2$$

$$\nabla \cdot A = 8$$

$$\nabla \times B = \left[ \begin{array}{ccc} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -3xz & 2xy \end{array} \right]$$

$$i \left| \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z} \right| - j \left| \frac{\partial}{\partial x} \quad \frac{\partial}{\partial z} \right| + k \left| \frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \right|$$

$$= 3xz \quad 2xy \quad yz \quad -3xz \quad 2xy \quad yz \quad -3xz$$

VISTALINE