Question 3

$A = x^2y^2 i + (xy^2 - y^2)j + xz^2 k$

$B = yz\,i - 3xz\,j + 2xy\,k$

$\phi = 3x^2y + xy^2z + 4y^2z^2 - 3$

$\nabla\phi = \left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right]\phi$

$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$

$\nabla\phi = i\left[y2+6xy\right] + j\left[3x^2 + xz - 8yz^2\right] + k\left[xy - 8y^2z\right]$

at point $(1,2,1)$

$\nabla\phi = i\left[2\times1+6\right] + j\left[3\times1^2 + 1\times1 - 8\times2\times1^2\right] + k\left[1\times2 - 8\times2^2\times1\right]$

$\nabla\phi = i(14) + j(-12) + k(-30)$

$\nabla\phi = 21 + 12j + 30k$

$\nabla\cdot A$

$\nabla = i\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$

$A = axi + ayj + azk$

$\nabla\cdot A = \left[i\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right]\cdot\left[axi + ayj + azk\right]$

$\nabla\cdot A = \frac{\partial}{\partial x}\cdot ax + \frac{\partial}{\partial y}\cdot ay + \frac{\partial}{\partial z}\cdot az$

$= 2xy + (x+z) + 2xz$

$\nabla\cdot A$ at $(1,2,1)$

$\nabla\cdot A = 2\times1\times2 + (1+1) + 2\times1\times1$

$\nabla\cdot A = 4 + 2 + 2$

$\nabla\cdot A = 8$

$$\nabla\times B = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -3xz & 2xy \end{vmatrix}$$

$$i\begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -3xz & 2xy \end{vmatrix} - j\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ yz & 2xy \end{vmatrix} + k\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ yz & -3xz \end{vmatrix}$$

$$i\left[\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(-3x^2)\right] - j\left[\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial z}(y^2)\right] + k\left[\frac{\partial}{\partial x}(-3x^2) - \frac{\partial}{\partial y}(y^2)\right]$$

$$= (x + 3x)i - j(xy - y) + k(-3z - z)$$

$\nabla \times B$ at $(1, 2, 1)$

$$\nabla \times B = (3 \times 1 + 3 \times 1)i - j(2 \times 2 - 2) + k(-3 \times 1 - 1)$$

$$\nabla \times B = (5)i - j(2) + k(-4)$$

$$\nabla \times B = 5i - 2j - 4k$$

Grad of div A

$$\nabla \cdot A = xy + (x + z) + 2x2$$

$$\nabla A = \left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right] \nabla \cdot A$$

$$\nabla A = [(2y + 0 + 2z)i + j(2x) + (1 + x)k]$$

$\nabla(\nabla \cdot A)$ at $(1, 2, 1)$

$$\nabla(\nabla \cdot A) = (2 \times 2 + 1 + 2 \times 1)i + (2 \times 1)j + (1 + 2 \times 1)k$$

$$= (7)i + (2)j + 3k$$

$$7i + 2j + 3k$$

Curl curl A

$$\text{curl } A = \nabla \times A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & (xy + yz) & (xz^2) \end{vmatrix}$$

$$i\begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (xy + yz) & (xz^2) \end{vmatrix} - j\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ x^2y & (xz^2) \end{vmatrix} + k\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x^2y & (xy + yz) \end{vmatrix}$$

$$i\left[\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy + yz)\right] - j\left[\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right]$$

$$+ k\left[\frac{\partial}{\partial x}(xy + yz) - \frac{\partial}{\partial y}(x^2y)\right]$$

$$i[0 - y] - j[z^2 - 0] + k[y - x^2]$$

at (1,2,1)

$\nabla \times A = -y\,i - z^2\,j + (y-x^2)k$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$i\left|\frac{\partial}{\partial y}(y-x^2) - \frac{\partial}{\partial z}(-z^2)\right| - j\left|\frac{\partial}{\partial x}(y-x^2) - \frac{\partial}{\partial z}(-y)\right| + k\left|\frac{\partial}{\partial x}(-z^2) - \frac{\partial}{\partial y}(-y)\right|$$

$$i(1+2z) - j(-2x+0) + k(0+1)$$

$\nabla \times (\nabla \times A)$ at (1,2,1)

$\nabla \times (\nabla \times A) = i(1+2\times 1) - j(-2\times 1) + k(1)$

$\nabla \times (\nabla \times A) = 3i + 2j + k$

Ujunwa Fatima
Computer Engineering
16/ENG01/057
ENG 252

1) Mathematical modelling is a description of a system using mathematical concepts and lan

Mathematical models defined as a process of developing a mathematical model.

methods of developing model in engineering system

1) Radioactivity
2) Mixing problems

Question 2

1) $r = (t^2+3t)i - 3\sin 3t\,j + 3e^{3t}k$

a) $\frac{dr}{dt} = (2t+3)i - (6\cos 3t)j + (6e^{2t})k$

b) $\frac{d^2r}{dt^2} = (2)i + (18\sin 3t)j + (12e^{2t})k$

$\frac{d^2r}{dt^2}$ at $t=0$ $= 2i + 18\sin 0\,j + 12e^{0}k$

$\frac{d^2r}{dt^2}$ at $t=0$ $= 2i + 12k$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{2^2+12^2}$

$= \sqrt{4+144}$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{148} = 12.2$ units