(iv) grad div A

grad $(2xy + (x+z) + 2xz)$

let $\text{div } A = C = \bar{\nabla} A$

$\bar{\nabla}(\bar{\nabla} A) = \bar{\nabla} C = i\frac{\partial C}{\partial x} + j\frac{\partial C}{\partial y} + k\frac{\partial C}{\partial z}$

$= i(2y + 1 + 2z) + j(2x) + k(1 + 2x)$

At $(1, 2, 1)$

$\bar{\nabla} C = i(2(2) + 1 + 2(1)) + j(2(1)) + k(1 + (2)(1))$

$= i(4 + 1 + 2) + j(2) + k(1 + 2)$

$= 7i + 2j + 3k$

(v) Curl curl A

$\text{Curl } A = \bar{\nabla} \times A$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & (xy + yz^2) & xz^2 \end{vmatrix}$$

$= i(0 - y) - j(z^2 - 0) + k(y - x^2)$

$= -yi - z^2 j + k(y - x^2)$

At $(1, 2, 1)$

$\text{Curl } A = -2i - j + k$

$\text{Curl curl } A = \bar{\nabla} \times (\bar{\nabla} \times A)$

$$\bar{\nabla} \times (\bar{\nabla} \times A) \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & -z^2 & (y - x^2) \end{vmatrix}$$

$= i(1 + 2z) - j(-2x - 0) + k(0 + 1)$

$= i(1 + 2z) + 2x^2 j + k$

At point $(1, 2, 1)$

$\nabla^2 \times (\bar{\nabla} \times A) = i(1 + 2(1)) + 2(1)^2 j + k$

$= 3i + 2j + k$

$\frac{\partial \phi}{\partial z} = xy - 8y^2z$

$\frac{\partial \phi}{\partial y} = 3x^2 + xz - 8yz^2$

At (1,2,1)

$\frac{\partial \phi}{\partial x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$

$\frac{\partial \phi}{\partial y} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3 + 1 - 16 = -12$

$\frac{\partial \phi}{\partial z} = (1)(2) - 8(2)^2(1) - 2 - 32 = -30$

$\nabla \phi = 14i - 12j - 30k$

(ii) $\nabla \cdot A = \frac{\partial a}{\partial x} + \frac{\partial ay}{\partial y} + \frac{\partial az}{\partial z}$

$A = axi + ayj + azk$

$\nabla \cdot A = 2xy + (x+z) + 2xz$

At (1,1,1)

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$= 4 + 2 + 2 = 8$

(iii) $\nabla \cdot B$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$= i(2x + 3x) - j(2y - y) + k(-3z - z)$

$= 5xi - yj - 4zk$

At (1,2,1)

$\nabla \times B = 5i - 2j - 4k$

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16/ENG05/022
ENG 282

Assignment

(a) A mathematical model is a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical and other terms.

(b) (i) Exponential growth / decay (use of ODE)
(ii) Mixing problems.

(2) $r = (t^2+3t)i - 2\sin 3t\,j + 3e^{2t}k$

(i) $\frac{\partial r}{\partial t} = (2t+3)i - 6\cos 3t\,j + 6e^{2t}k$

(ii) $\frac{\partial^2 r}{\partial t^2} = 2i + 18\sin 3t\,j + 12e^{2t}k$

(iii) $\left.\frac{\partial^2 r}{\partial t^2}\right|_{t=0} = 2i + 12k$

$\left|\frac{\partial^2 r}{\partial t^2}\right| = \sqrt{2^2+17^2} = \sqrt{4+144} = \sqrt{148}$

$= 2\sqrt{37} = 12.7.$

(3) $A = x^2y\,i + (xy+y^2)j + xz^2k$

$B = yz\,i - 3xz\,j + 2xy\,k$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

(i) $\nabla\phi = \frac{\partial d i}{\partial x} + \frac{\partial d j}{\partial y} + \frac{\partial d k}{\partial z}$

$\frac{\partial \phi}{\partial x} = 6xy + y^2$