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MAT. NO: 17/ENG01/032
DEPT: CHEMICAL
COURSE: ENG282 Assignment 3.

1) Mathematical models is defined as a process of developing a ~~mathem~~ mathematical model.

Methods of developing model in engineering system.

1, Radioactivity
2, Mixing problems.

2) · Question 2.

i) $r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t}k$

a) $\frac{dr}{dt} = (2t+3)i - (6\cos 3t)j + (6e^{2t})k$

b) $\frac{d^2r}{dt^2} = (2)i + (18\sin 3t)j + (12e^{2t})k$

$\left.\frac{d^2r}{dt^2}\right|_{at\, t=0} = 2i + 18\sin 0\, j + 12e^{0}k.$

$\left.\frac{d^2r}{dt^2}\right|_{at\, t=0} = 2i + 12k.$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{2^2+12^2}$

$= \sqrt{4+144}.$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{148}$

$= 10.2$ units.

Question 3.

$A = x^2 y i + (xy + yz) j + xz^2 k$

$B = yz i - 3xz j + 2xy k$

$\phi = 3x^2 y + xyz + 4y^2 z^2 - 3$

$$\nabla\phi = \left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \right]\phi$$

$$\nabla\phi = i\left[\frac{\partial \phi}{dx} + j\frac{\partial \phi}{dy} + k\frac{\partial \phi}{\partial z}\right]$$

$$\nabla\phi = i(yz + 6xy + j(3x^2 + xz - 8yz^2) + k(xy - 8y^2 z)$$

at point $(1,2,1)$

$$\nabla\phi = 1[2\times1 + 12] + y[3\times 1^2 + 1\times1 - 8\times2\times1^2] + k(1\times2 - 8\times2^2\times1]$$

$$\nabla\phi = 1(14) + j(-12) + k(-30)$$

$$\nabla\phi = 21 + 12j + 30k.$$

$\nabla \cdot A$

$$\nabla = i\frac{\partial}{dx} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

$$A = ax\,i + ay\,j + az\,k$$

$$\nabla\cdot A = \left[i\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right]\cdot\left[ax\,i + ay\,j + az\,k\right]$$

$$\nabla\cdot A = \frac{\partial}{\partial x}\cdot ax + \frac{\partial}{\partial y}\cdot ay + \frac{\partial}{\partial z}\cdot az$$

$$= 2xy + (x+z) + 2xz$$

$\nabla \cdot A$ at $(1,2,1)$

$$\nabla\cdot A = 2\times1\times2 + (1+1)\times2\times1\times1$$

$$\nabla\cdot A = 4 + 2 + 2$$

$$\nabla\cdot A = 8.$$

$$\nabla\times B = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$$

$$i\begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -3xz & 2xy \end{vmatrix} \; y\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ yz & 2xy \end{vmatrix} + k\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ y^2 & -3xz \end{vmatrix}$$

$$i\left(\frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial z}(-3xz)\right] - y\left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(yz)\right] +$$

$$k\left[\frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}(y^2)\right]$$

$= (2x + 3x)i - y(2y - y) + k(-3z - z)$

$\nabla \times B$ at $(1,2,1)$

$\nabla \times B = (2\times1 + 3\times1)i \; y(2\times2 - 2) + k(-3\times1 - 1)$

$\nabla \times B = (5)i - j(2) + k(-4)$

$\nabla \times B = 5i - 2j - 4k$

Grad of div A

$\nabla \cdot A = 2xy + (x+2) + 2xz$

$\nabla A = \left[i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right] \nabla \cdot A$

$= \left[(2y + 1 + 2z)i + j(2x) + (1 + 2x)k\right]$

$\nabla(\nabla \cdot A)$ at $(1,2,1)$

$\nabla(\nabla \cdot A) = (2\times2 + 1 + 2\times1)i + (2\times1)j + (1 + 2\times1)k$

$= (7)i + (2)j + 3k$

$= 7i + 2j + 3k$ //.

Calculate A

$\text{Curl } A = \nabla \times A \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & (xy + y^2) & (xz^2) \end{vmatrix}$

$$i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy+y^2) & (xz^2) \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2y & (xz^2) \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2y & (xy+y^2) \end{vmatrix}$$

$$i\left(\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy+y^2)\right) - j\left[\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right]$$

$$+k\left[\frac{\partial}{\partial x}(xy+yz) - \frac{\partial}{\partial y}(x^2y)\right]$$

$$i[0-y] - j[z^2-0] + k[y-x^2]$$

$$\nabla \times A = -yi - z^2j + (y-x^2)k$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$i\left|\frac{\partial}{\partial y}(y-x^2) - \frac{\partial}{\partial z}(-z^2)\right| - j\left(\frac{\partial}{\partial x}(y-x^2) - \frac{\partial}{\partial z}(-y)\right)$$

$$+k\left[\frac{\partial}{\partial x}(-z^2) - \frac{\partial}{\partial y}(-y)\right]$$

$$\hat{i}(1+z^2) - j(-2x+0) + k(0+1)$$

$\nabla \times (\nabla \times A)$ at $(1,2,1)$

$$\nabla \times (\nabla \times A) = i(1\times 2\times 1) - j(-2\times 1) + (1)$$

$$\nabla \times (\nabla \times A) = 3i + 2j + k$$

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