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16/ENG0510/17

Mechatronics

ENG 282

### Assignment III

1a) Mathematical modelling can be defined as the process of setting up a model of an engineering problem, solving it mathematically and interpreting the result in physical or other terms. The model is the formulation of the problem as a mathematical expression in terms of variables, functions and equations.

(ii) Two Methods of obtaining models for engineering systems

a) Matthu's Law

b) Exponential growth and decay

$$2 \quad r = (t^2 + 3t)i - 2\sin 3tj + 3e^{2t}k$$

$$(i) \quad \frac{dr}{dt} = (2t + 3)i - 6\cos 3tj + 6e^{2t}k$$

$$(ii) \quad \frac{d^2r}{dt^2} = 2i + 18\sin 3tj + 12e^{2t}k$$

$$(iii) \quad \text{at } t=0, \quad \frac{d^2r}{dt^2} = 2i + 18\sin 3(0)j + 12e^{2(0)}k$$

$$\begin{aligned} \left| \frac{d^2r}{dt^2} \right|_{t=0} &= \sqrt{2^2 + 12^2} \\ &= \sqrt{4 + 144} \\ &= \sqrt{148} = 12.17 \end{aligned}$$

$$\left| \frac{d^2r}{dt^2} \right|_{t=0} = 12.17 //$$



$$3 \quad A = x^2 y i + (x y + y z) j + x z^2 k$$

$$B = y z i - 3 x z j + 2 x y k$$

$$\phi = 3 x^2 y + x y z - 4 y^2 z^2 - 3$$

at the point (1, 2, 1)

$$(i) \quad \nabla \phi = \text{grad } \phi = \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) \cdot (3 x^2 y + x y z - 4 y^2 z^2 - 3)$$

$$= \frac{\partial}{\partial x} (3 x^2 y + x y z - 4 y^2 z^2 - 3) i + \frac{\partial}{\partial y} (3 x^2 y + x y z - 4 y^2 z^2 - 3) j + \frac{\partial}{\partial z} (3 x^2 y + x y z - 4 y^2 z^2 - 3)$$

$$= i (6 x y + y z) + j (3 x^2 + x z - 8 y^2 z) + k (x y - 8 y^2 z)$$

at the point (1, 2, 1)

$$\nabla \phi = i [6(1)(2) + (2)(1)] + j [3(1)^2 + (1)(1) - 8(2)(1)^2] + k [(1)(2) - 8(2)^2(1)]$$

$$= i (12 + 2) + j (3 + 1 - 16) + k (2 - 32)$$

$$= 14 i - 12 j - 30 k$$

$$(ii) \quad \text{div } A = \nabla \cdot A = \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) \cdot (x^2 y i + (x y + y z) j + x z^2 k)$$

$$\nabla \cdot A = \frac{\partial}{\partial x} (x^2 y) + \frac{\partial}{\partial y} (x y + y z) + \frac{\partial}{\partial z} (x z^2)$$

$$= 2 x y + (x + z) + 2 x z$$

at (1, 2, 1)

$$= 2(1)(2) + (1 + 1) + 2(1)(1)$$

$$= 4 + 2 + 2$$

$$\nabla \cdot A = 8$$

$$(ii) \quad \text{Curl } B = \nabla \times B = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y z & -3 x z & 2 x y \end{vmatrix}$$

$$\nabla \times B = i \left[ \frac{\partial}{\partial y} (2 x y) + \frac{\partial}{\partial z} (3 x z) \right] - j \left[ \frac{\partial}{\partial x} (2 x y) - \frac{\partial}{\partial z} (y z) \right] + k \left[ \frac{\partial}{\partial x} (-3 x z) - \frac{\partial}{\partial y} (y z) \right]$$

$$= i (2 x + 3 x) - j (2 y - y) + k (-3 z - z)$$

$$= 5 x i - y j - 4 z k$$



iv  $\text{grad div } A = \nabla(\nabla \cdot A)$

$$\nabla \cdot A = \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) \cdot x^2y i + (xy + yz) j + xz^2 k$$

$$= \frac{\partial}{\partial x} (x^2y) + \frac{\partial}{\partial y} (xy + yz) + \frac{\partial}{\partial z} (xz^2)$$

$$\nabla \cdot A = 2xy + (x + z) + 2xz$$

$$\nabla(\nabla \cdot A) = \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) \cdot 2xy + (x + z) + 2xz$$

$$= \frac{\partial}{\partial x} (2xy + (x + z) + 2xz) i + \frac{\partial}{\partial y} (2xy + (x + z) + 2xz) + \frac{\partial}{\partial z} (2xy + (x + z) + 2xz) k$$

$$= i(2y + 1 + 2z) + 2x j + (1 + 2x) k$$

at (1, 2, 1)

$$\nabla(\nabla \cdot A) = [(2(2) + 1 + 2(1))] i + [2(1)] j + [1 + 2(1)] k$$

$$= (4 + 1 + 2) i + 2j + (1 + 2) k$$

$$= 7i + 2j + 3k //$$

v  $\text{Curl Curl } A = \nabla \times (\nabla \times A)$

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & (xy + yz) & xz^2 \end{vmatrix}$$

$$= i \left[ \frac{\partial}{\partial y} (xz^2) - \frac{\partial}{\partial z} (xy + yz) \right] - j \left[ \frac{\partial}{\partial x} (xz^2) - \frac{\partial}{\partial z} (x^2y) \right] + k \left[ \frac{\partial}{\partial x} (xy + yz) - \frac{\partial}{\partial y} (x^2y) \right]$$

$$\nabla \times A = (-y) i - z^2 j + (y - x^2) k$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & -z^2 & (y - x^2) \end{vmatrix}$$

$$= i \left[ \frac{\partial}{\partial y} (y - x^2) + \frac{\partial}{\partial z} (z^2) \right] + j \left[ \frac{\partial}{\partial x} (y - x^2) + \frac{\partial}{\partial z} (-y) \right] + k \left[ \frac{\partial}{\partial x} (-z^2) + \frac{\partial}{\partial y} (-y) \right]$$

$$\nabla \times (\nabla \times A) = i(1 + 2z) - j(-2x) + k(1)$$



$$= (1+2z)i + 2xzj + k$$

at  $(1, 2, 1)$

$$\nabla \times (\nabla \times A) = (1+2(1))i + j(1+2(1)) + k \\ = 3i + 2j + k //$$