Assignment III

1(i) Mathematical modelling can be defined as the process of setting up a model of an engineering problem, solving it mathematically and interpreting the result in physical or other terms. The model is the formulation of the problem as a mathematical expression in terms of variables, functions and equations.

(ii) Two Methods of obtaining models for engineering systems

a) Malthu's Law

b) Exponential growth and decay

2 $r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t}k$

(i) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t}k$

(ii) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t}k$

(iii) at $t=0$, $\frac{d^2r}{dt^2} = 2i + 18\sin 3(0)j + 12e^{2(0)}k$

$= 2i + 12k$

$$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{2^2+12^2}$$

$$= \sqrt{4+144}$$

$$= \sqrt{148} = 12.17$$

$$\left|\frac{d^2r}{dt^2}\right|_{t=0} = 12.17$$

3 $A = x^2y i + (xy + yz) j + xz^2 k$

$B = yz i - 3xz j + 2xy k$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

at the point $(1,2,1)$

(i) $\nabla\phi = \text{grad}\,\phi = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot (3x^2y + xyz - 4y^2z^2 - 3)$

$= \frac{\partial}{\partial x}(3x^2y + xyz - 4y^2z^2 - 3)i + \frac{\partial}{\partial y}(3x^2y + xyz - 4y^2z^2 - 3)j + \frac{\partial}{\partial z}(3x^2y + xyz - 4y^2z^2 - 3)$

$= i(6xy + yz) + j(3x^2 + xz - 8yz^2) + k(xy - 8y^2z)$

at the point $(1,2,1)$

$\nabla\phi = i[6(1)(2) + (2)(1)] + j[3(1)^2 + (1)(1) - 8(2)(1)^2] + k[(1)(2) - 8(2)^2(1)]$

$= i(12 + 2) + j(3 + 1 - 16) + k(2 - 32)$

$= 14i - 12j - 30k$

(ii) $\text{div}\,A = \nabla \cdot A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot (x^2yi + (xy + yz)j + xz^2k)$

$\nabla \cdot A = \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(xy + yz) + \frac{\partial}{\partial z}(xz^2)$

$= 2xy + (x + z) + 2xz$

at $(1,2,1)$

$= 2(1)(2) + (1 + 1) + 2(1)(1)$

$= 4 + 2 + 2$

$\nabla \cdot A = 8$

ii Curl $B = \nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$

$\nabla \times B = i\left[\frac{\partial}{\partial y}(2xy) + \frac{\partial}{\partial z}(3xz)\right] - j\left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(yz)\right] + k\left[\frac{\partial}{\partial x}(3xz) - \frac{\partial}{\partial y}(yz)\right]$

$= i(2x + 3x) - j(2y - y) + k(-3z - z)$

$= 5xi - yj - 4zk$

Scanned by CamScanner

iv $\text{grad div } A = \nabla(\nabla \cdot A)$

$$\nabla \cdot A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot x^2y\, i + (xy+yz)j + xz^2k$$

$$= \frac{\partial}{\partial x}(x^2y) + \frac{\partial}{\partial y}(xy+yz) + \frac{d}{\partial z}(xz^2)$$

$$\nabla \cdot A = 2xy + (x+z) + 2xz$$

$$\nabla(\nabla \cdot A) = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot 2xy + (x+z) + 2xz$$

$$= \frac{\partial}{\partial x}(2xy + (x+z) + 2xz)i + \frac{\partial}{\partial y}(2xy + (x+z) + 2xz) + \frac{\partial}{\partial z}(2xy + (x+z) + 2xz)k$$

$$= i(2y+1+2z) + 2x\, j + (1+2x)k$$

at $(1,2,1)$

$$\nabla(\nabla \cdot A) = [(2)(2)+1+2(1)]i + [2(1)]j + [1+2(1)]k$$

$$= (4+1+2)i + 2j + (1+2)k$$

$$= 7i + 2j + 3k$$

v $\text{Curl Curl } A = \nabla \times (\nabla \times A)$

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial y \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy+yz)\right] - j\left[\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right] + k\left[\frac{\partial}{\partial x}(xy+yz) - \frac{\partial}{\partial y}(x^2y)\right]$$

$$\nabla \times A = (-y)i - z^2j + (y-x^2)k$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(y-x^2) + \frac{\partial}{\partial z}(z^2)\right] + i\left[\frac{\partial}{\partial x}(y-x^2) + \frac{\partial}{\partial z}(y)\right] + k\left[\frac{\partial}{\partial x}(-z^2) + \frac{\partial}{\partial y}(y)\right]$$

$$\nabla \times (\nabla \times A) = i(1+2z) - j(-2x) + k(1)$$

$= (1+2z)i + 2xj + k$

at $(1,2,1)$

$\nabla \times (\nabla \times A) = (1+2(1)) + j\,(+2(1)) + k$

$= 3i + 2j + k$