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Elect/Elect
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Ass.

1) A mathematical model is a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical and other terms

2) I) Exponential Growth Decay (use of ODE)
II) Mixing Problems

2) $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t}k$

i) $dr/dt = (2t+3)i - 6\cos 3t j + 6e^{2t}k$

ii) $d^2r/dt^2 = 2i + 18\sin 3t j + 12e^{2t}k$

iii) $d^2r/dt^2 \Big|_{t=0} = 2i + 12k$

$\left| \frac{d^2 r}{dt^3} \right| = \sqrt{2^2 + 12^2} = \sqrt{4+144}$
$\sqrt{148} = 2\sqrt{37} = 12.17$

3) $A = x^2yi + (xy + y^2)j + xz^2k$
$B = yzi - 3xzj + 2xyk$
$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

I) $\nabla\phi = \frac{d\phi}{dx} i + \frac{d\phi}{dy} j + \frac{d\phi}{dz} k$

$\frac{d\phi}{dx} = 6xy + yz$

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$$\frac{\partial \phi}{\partial y} = 3x^2 + xz - 8yz^2$$

At $(1,2,1)$

$$\frac{\partial \phi}{\partial x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$$

$$\frac{\partial \phi}{\partial y} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3+1-16$$
$$= -12$$

$$\frac{\partial \phi}{\partial z} = (1)(2) - 8(2)^2(1) = 2 - 32$$
$$= -30$$

$$\nabla \phi = 14i - 12j - 30k$$

ii) $\nabla \cdot A = \frac{\partial a}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$A = a_x i + a_y j + a_z k$

$\nabla \cdot A = 2xy + (x + z) + 2xz$

At $(1,2,1)$

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$= 4 + 2 + 2 = 8$

iii) $$\nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & 3xz & 2xy \end{vmatrix}$$

$= i(2x + 3x) - j(2y - y) + k(-3z - z)$

$= 5xi - yj - 4zk$

At $(1,2,1)$

$\nabla \times B = 5i - 2j - 4k$

iv) grad div A

grad $(2xy + (x + z) + 2xz)$

let div $A = C = \nabla A$

$$\nabla(\nabla \cdot A) = \nabla c = i\frac{\partial c}{\partial x} + j\frac{\partial c}{\partial y} + k\frac{\partial c}{\partial z}$$

$$= i(2y + 1 + 2z) + j(2x) + k(1 + 2x)$$

At $(1, 2, 1)$

$$\nabla c = i[2(2) + 1 + 2(1)] + j[2(1)] + k[1 + (2)(1)]$$

$$= i(4 + 1 + 2) + j(2) + k(1 + 2)$$

$$= 7i + 2j + 3k$$

v Curl ~~Cu~~ A

$$\text{Curl } A = \nabla \times A$$

$$= \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dx} & \frac{d}{dz} \\ x^2 y & (xy + yz) & xz^2 \end{vmatrix}$$

$$= i(0 - y) - j(z^2 - 0) + k(y - x^2)$$

$$= -yi - z^2 j + k(y - x^2)$$

At $(1, 2, 1)$

$$\text{Curl } A = -2i \cancel{\phi} - j + k$$

$$\text{Curl } A = \nabla \times (\nabla \times A)$$

$$\nabla \times (\nabla \times A) \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -y & -z^2 & (y - x^2) \end{vmatrix}$$

$$= i(1 + 2z) - j(-2x - 0) + k(0 + 1)$$

$$= i(1 + 2z) + 2x^2 j + k$$

At point $(1, 2, 1)$

$$\nabla \times (\nabla \times A) = i(1 + 2(1)) + 2(1)^2 j + k$$

$$= 3i + 2j + k$$