ENG 282

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1(a) A mathematical model is a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical and other terms.

(b) i, Mixing problems.
(ii) Exponential growth / decay (use of ODE).

2) $r = (t^2+3t)\hat{i} - 2\sin 3t\,\hat{j} + 3e^{2t}\,\hat{k}$

i) $\frac{dr}{dt} = (2t+3)\hat{i} - 6\cos 3t\,\hat{j} + 6e^{2t}\hat{k}$

ii, $\frac{d^2r}{dt^2} = 2\hat{i} + 18\sin 3t\,\hat{j} + 12e^{2t}\hat{k}$

iii) $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2\hat{i} + 12\hat{k}$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2+12^2}$

$= \sqrt{4+144}$

$= \sqrt{148}$

$= 2\sqrt{37}$

$= \underline{\underline{12.17}}$

(3) $A = x^2yi + (xy+yz)j + xz^2k$

$B = yzi - 3xzj + 2xyk$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

$\therefore \nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$

$\frac{\partial\phi}{\partial x} = 6xy + yz$

$\frac{d\phi}{dz} = xy - 8y^2z$

$\frac{\partial\phi}{\partial y} = 3x^2 + xz - 8yz^2$

At $(1, 2, 1)$

$\frac{\partial\phi}{\partial x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$

$\frac{\partial\phi}{\partial y} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3 + 1 - 16 = -12$

$\frac{\partial\phi}{\partial z} = (1)(2) - 8(2)^2(1) = 2 - 32 = -30$

$\nabla\phi = 14\hat{i} - 12\hat{j} - 30\hat{k}$

$\therefore \nabla\cdot A = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$A = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$\nabla\cdot A = 2xy + (x+2) + 2xz$

At $(1, 1, 1)$

$\nabla\cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$= 4 + 2 + 2$

$= 8$

(iii) $\nabla \times B$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$$= \hat{i}(2x+3x) - \hat{j}(2y-y) + \hat{k}(-3z-z)$$

$$= 5x\hat{i} - y\hat{j} - 4z\hat{k}$$

At $(1,2,1)$

$$\nabla \times B = 5\hat{i} - 2\hat{j} - 4\hat{k}$$

iv) grad div A

grad $[2xy + (x+z) + 2xz]$

Let div $A = C = \nabla A$

$$\nabla(\nabla A) = \nabla C = \hat{i}\frac{\partial c}{\partial x} + \hat{j}\frac{\partial c}{\partial y} + \hat{k}\frac{\partial c}{\partial z}$$

$$= \hat{i}(2y+1+2z) + \hat{j}(2x) + \hat{k}(1+2x)$$

At $(1,2,1)$

$$\nabla C = \hat{i}(2(2)+1+2(1)) + \hat{j}(2(1)) + \hat{k}(1+(2)$$

$$= 7\hat{i} + 2\hat{j} + 3\hat{k}$$

(v) Curl Curl A

Curl A $= \nabla \times A$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= \hat{i}(0-y) - \hat{j}(z^2-0) + \hat{k}(y-x^2)$$

$$= -y\hat{i} - z^2\hat{j} + \hat{k}(y-x^2)$$

At $(1,2,1)$

Curl $A = -2\hat{i} - \hat{j} + \hat{k}$

Curl Curl $A = \nabla \times (\nabla \times A)$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= \hat{i}(1+2z) - \hat{j}(-2x^2-0) + \hat{k}(0+1)$$

$$= \hat{i}(1+2z) + 2x^2\hat{j} + \hat{k}$$

At points $(1, 2, 1)$

$$\nabla \times (\nabla \times A) = \hat{i}\,(1+2(1)) + 2(1)^2\hat{j} + \hat{k}$$

$$= 3\hat{i} + 2\hat{j} + \hat{k}$$