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16/Eng06/011
Mechanical Engineering
Eng 282

1 Mathematical modelling
This is the process of setting up a model, solving it mathematically, and interpreting the result in physical or in order terms

ii using balance law - law of conservation of mass
b Forming a differential equation from an existing algebraic equation of the system.

2 $r = (t^2 + 3t)i - 2\sin 3t\, j + 3e^{2t} k$

$\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t} k.$

$\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right)$

$\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t} k.$

$\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0)\, j + 12e^{2(0)} k.$

$=$

$2i + 18\sin 0\, j + 12e^0 k$

$2i + 18 \times 0 j + 12 \times 1 k$

$= 2i + 12k$

iii $\left|\frac{d^2r}{dt^2}\right| = |2i + 12k|$

$=$

$\sqrt{(2i)^2 + (12k)^2}$

$=$

$\sqrt{4 \times i \cdot i + 144 \times k \cdot k}$ $= \sqrt{148}$

$= 2\sqrt{37}$

3) at Point $(1, 2, 1)$

$A = x^2 y i + (xy + yz) j + xz^2 k$.

$B = yz i - 3xz j + 2xy k$.

$\phi = 3x^2 y + xyz - 4y^3 z^2$

$$\bar{\nabla}\phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k$$

$$\bar{\nabla}\phi = \frac{\partial (3x^2 y + xyz - 4y^2 z^2 - 3)}{\partial x} i + \frac{\partial (3x^2 y + xyz - 4y^2 z^2 - 3)}{\partial y} j + \frac{\partial (3x^2 y + xyz - 4y^2 z^2 - 3)}{\partial z} k$$

$$\bar{\nabla}\phi = (6xy + yz) i + (3x^2 + xz - 8yz^2) j + (xy - 8y^2 z) k.$$

at point $1, 2, 1$

$$\bar{\nabla}\phi = (6(1)(2) + (1)(1)) i + (3(1^2) + (1)(1) - 8(2)(1^2)) j + ((1)(2) - 8(2^2)(1)) k$$

$$\bar{\nabla}\phi_{(1,2,1)} = 14i - 12j - 30k$$

ii) $\bar{\nabla} \cdot A = \frac{\partial}{\partial x} A i + \frac{\partial}{\partial y} \cdot A j + \frac{\partial}{\partial z} \cdot A k$

$\bar{\nabla} A = 2xy \, i \cdot i + (x + z) j \cdot j + 2xz \, k \cdot k$

$= 2xy + (x + z) + 2xz$

$\bar{\nabla} A (1, 2, 1) = 2(1)(2) + (1) + (1) + 2(1)(1)$

$4 + 2 + 2 = 8$

$\bar{\nabla} A = 8$

iii) $$\bar{\nabla} \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial y} \cdot 2xy - \frac{\partial}{\partial z} - 3xz\right) i - j \left(\frac{\partial}{\partial x} \cdot 2xy - \frac{\partial}{\partial z} \cdot yz\right) + k$$

$$\left(\partial/\partial x \cdot -3z - \partial/\partial y \cdot yz\right)$$

$\nabla \times B = (2x - (-3x))i - j(2y - y) + k(-3z - z)$

$\nabla \times B = 5xi - yj - k(4z)$

$\nabla \times B = 5xi - yj - 4zk.$

$1, 2, 1 = 5(1)i - (2)j - 4(1)k$

$= 5i - 2j - 4k$

rul

$\text{grad} \cdot \text{div} A = \nabla \cdot \nabla A$

$= \frac{\partial}{\partial x} \cdot \nabla A i + \frac{\partial}{\partial y} \cdot \nabla A j + \frac{\partial}{\partial z} \cdot \nabla A k$

recall $\nabla A = 2xy + x + z + 2xz$

$\nabla \cdot \nabla A = \frac{\partial}{\partial x}(2xy + x + z + 2xz) \cdot i + \frac{\partial}{\partial y}(2xy + x + z + 2xz)j + \partial/\partial z (2xy + x + z + 2xz)k$

$\nabla \cdot \nabla A = (2y + 1 + 2z) \cdot i + (2x)j + (1 + 2x)k$

at point $(1, 2, 1)$

$= 2(2) + 1 + 2(1)i + 2(1)j + 1 + 2(1)k$

$(4 + 1 + 2)i + 2j + (1 + 2)k$

$\nabla \cdot \nabla A = 7i + 2j + 3k$

$\text{grad div } A = 7i + 2j + 3k$

$\text{Curl curl } A = \text{curl}(\text{curl } A)$

$$\text{Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & (xy + yz) & xz^2 \end{vmatrix}$$

$$\text{Curl } A = i \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy + yz) & xz^2 \end{vmatrix} - j \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2 y & xz^2 \end{vmatrix} + k \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2 y & (xy + yz) \end{vmatrix}$$

Curl A

$$= i\left(\frac{\partial}{\partial y} \cdot xz^2 - \frac{\partial}{\partial z}(xy + yz)\right) - j\left(\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2 y)\right)$$

$$+k\left(\frac{\partial}{\partial x}(xy+yz)-\frac{\partial}{\partial y}(x^2y)\right)$$

$$= i(0-y) + -j(z^2-0) + k(y-x^2)$$

$$\text{Curl } A = -yi - z^2 j + (y-x^2)k$$

$$\text{Curl } A = -yi - z^2 j + (y-x^2)k$$

$$\text{Curl Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y_i & -z^2 & y-x^2 \end{vmatrix}$$

$$i\left(\frac{\partial}{\partial y}\cdot(y-x^2)-\frac{\partial}{\partial z}-z^2\right)-j\left(\frac{\partial}{\partial x}\cdot y-x^2-\frac{\partial}{\partial z}\cdot y\right)$$

$$+k\left(\frac{\partial}{\partial x}\cdot z^2-\frac{\partial}{\partial y}\cdot(-y)\right)$$

Curl Curl A

$$= (1+2z)i - j(-2x-0) + k(0-(-1))$$

$$\text{curl curl } A = (1+2z)i + 2xj + k$$

$$\text{Curl curl } A = (1+2z)i + 2xj + k$$

$$= (1+2(1))i + 2(1)j + k$$

$$= 3j + 2j + k = 3i + 2j + k$$

$$\text{Curl Curl } A = 3i + 2j + k$$