

1. Mathematical modelling is the process of using various mathematical structures (i.e. graphs, equations, diagram etc.) to represent real world situations. The model provides an abstraction that reduces a problem to its essential characteristics.

1bi Using the balanced law

i. By forming differential equations from an algebraic equation of the system

$$2. \quad r = (f^2 + 3t)i - 2\sin 3t j + 3e^{3t} k$$

$$\frac{dr}{dt} = (2i + 32i + 6\cos 3t j + 6e^{3t} k$$

$$\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{3t} k$$

$$at \quad t = 0$$

$$\frac{d^2r}{dt^2} = 2i + 12k$$

$$\frac{d^2r}{dt^2}$$

$$\sqrt{f^2 + 12^2} \text{ where } t = 0$$

$$\therefore \sqrt{2^2 + 12^2} = \sqrt{4+144} = \sqrt{148} = 12 \cdot 17$$

$$3. \quad A = x^2 y i + (xy + yz) j + xz^2 k$$

$$B = yz i - 3xz j + 2xyz k$$

$$\phi = 3x^2y + 2xy - 4y^2z^2 - 3$$

8th m

$$\nabla \phi = (6xy + yz)i + (3x^2 + xz - 2y^2z)j + (xy - 8yz^2)k$$

$$\nabla \cdot A = (x^2y)i \cdot \frac{d}{dx}x + (xy + yz)j \cdot \frac{d}{dy}y + (xz)k \cdot \frac{d}{dz}k$$

$$i \cdot i = f.y + k.k$$

$$2xy + xz + 2yz^2$$

$$\nabla \cdot AB = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$= i \left(\frac{d}{dy} (2xy) - \frac{d}{dz} (+3xz) \right) - j \left(\frac{d}{dz} (2xy) \right)$$

$$- k \left(\frac{d}{dx} (yz) + k \left(\frac{d}{dx} (+3xz) - \frac{d}{dy} (yz) \right) \right)$$

$$= i(2xy + 3x) - j(2y - y) + k(-3z - 2)$$

$$= 5xi - 4yj - 4zk$$

$\text{grad } \nabla \cdot A$

$$\nabla \cdot A = x^2y \cdot \frac{d}{dx} + (xy + yz) y \cdot \frac{d}{dy} + xz^2 k \cdot \frac{d}{dz} k$$

$$= (i \cdot i + j \cdot j + k \cdot k)$$

$$= 2xy + xz + 2yz^2$$

$$\nabla(\nabla \cdot A) = (2xy + xz + 2yz^2) \frac{d}{dx} + (2xy + xz + 2yz^2) \frac{d}{dy} + (2xy + xz + 2yz^2) \frac{d}{dz} k$$

$$= (2y + 1 + 2z)i + (2x^2)j + (2x + 1)k$$

Curl curl A.

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & (xy+z^2) & xz \end{vmatrix}$$

$$(d/dy(xz)) - d/dz(xy + z^2) \cdot j (d/dx(xz)) + k$$

$$+ k (d/dx(xy + z^2)) - d/dy(x^2y)$$

$$= -z^2j + (y - x^2)k$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= i \left(\frac{\partial}{\partial y} (y - x^2) - \frac{\partial}{\partial z} (-z^2) \right) - j \left(\frac{\partial}{\partial z} (y - x^2) - \frac{\partial}{\partial x} (-y) \right) - k \left(\frac{\partial}{\partial x} (-z^2) - \frac{\partial}{\partial y} (-y) \right)$$