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Department: Mechatronics Engineering

ENG 282

Assignment III

Solution

Question 1:

i) Mathematical modelling is the process of setting up ~~the~~^a model for an engineering problem, solving it mathematically and interpreting the results in physical or other terms.

ii) \rightarrow Mathematical
 \rightarrow Physical

Question 2

$$r = (t^2 + 3t)i - (2\sin 3t)j + (3e^{2t})k$$

i) $\frac{dr}{dt}$

$$\frac{dr}{dt} = (2t + 3)i - (6\cos 3t)j + (6e^{2t})k$$

ii) $\frac{d^2r}{dt^2}$

$$\frac{d^2r}{dt^2} = 2i - (-6 \cdot 3 \sin 3t)j + (6 \cdot 2e^{2t})k$$

$$\frac{d^2r}{dt^2} = 2i + (18 \sin 3t)j + 12e^{2t}k$$

iii) $\left. \frac{d^2r}{dt^2} \right|_{t=0} = 2i + 18 \sin 3(0)j + 12e^{2(0)}k$

$$\therefore \left. \frac{d^2r}{dt^2} \right|_{t=0} = 2i + 12k$$

Question 3:

$$A = x^2y \mathbf{i} + (xy + yz) \mathbf{j} + xz^2 \mathbf{k}$$

$$B = yz \mathbf{i} - 3xz \mathbf{j} + 2xy \mathbf{k}$$

$$\phi = 3xy + xyz - 4y^2z^2 - 3$$

at $(1, 2, 1)$.

(i) $\nabla \phi$.

$$\nabla \phi = \left(\mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z} \right) (3xy + xyz - 4y^2z^2 - 3)$$

$$\nabla \phi = \mathbf{i} \frac{\partial}{\partial x} (3xy + xyz - 4y^2z^2 - 3) + \mathbf{j} \frac{\partial}{\partial y} (3xy + xyz - 4y^2z^2 - 3) + \mathbf{k} \frac{\partial}{\partial z} (3xy + xyz - 4y^2z^2 - 3)$$

$$\nabla \phi = \mathbf{i} (3y + yz) + \mathbf{j} (3x + xz - 8yz^2) + \mathbf{k} (xy - 8y^2z)$$

$$\nabla \phi \Big|_{(1,2,1)} = [6(1)(2) + (2)(1)] \mathbf{i} + [3(1)^2 + (1)(1) - 8(2)(1)^2] \mathbf{j} + [(1)(2) - 8(2)^2] \mathbf{k}$$

$$\nabla \phi \Big|_{(1,2,1)} = 14 \mathbf{i} - 12 \mathbf{j} - 30 \mathbf{k}$$

$$\therefore \nabla \phi \Big|_{(1,2,1)} = 14 \mathbf{i} - 12 \mathbf{j} - 30 \mathbf{k}$$

(ii) $\nabla \cdot A$

$$\nabla \cdot A = \left(\mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z} \right) (x^2y \mathbf{i} + (xy + yz) \mathbf{j} + xz^2 \mathbf{k})$$

$$\nabla \cdot A = \frac{\partial}{\partial x} (x^2y) \mathbf{i} \cdot \mathbf{i} + \frac{\partial}{\partial y} (xy + yz) \mathbf{j} \cdot \mathbf{j} + \frac{\partial}{\partial z} (xz^2) \mathbf{k} \cdot \mathbf{k}$$

$$\nabla \cdot A = \frac{\partial}{\partial x} (x^2y) + \frac{\partial}{\partial y} (xy + yz) + \frac{\partial}{\partial z} (xz^2)$$

$$\nabla \cdot A = 2xy + (x + z) + 2xz$$

$$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$$

$$\left. \vphantom{\nabla \cdot A} \right|_{(1,2,1)}$$

$$\nabla \cdot A \Big|_{(1,2,1)} = 4 + 2 + 2 = 8$$

$$\nabla \cdot A = 8$$

$$\left. \vphantom{\nabla \cdot A} \right|_{(1,2,1)}$$

iii) $\nabla \times B$

$\nabla \times B =$	+	-	+
	i	j	k
	$\frac{\partial}{\partial x}$	$\frac{\partial}{\partial y}$	$\frac{\partial}{\partial z}$
	yz	$-3xz$	$2xy$

$\nabla \times B =$	i	$\frac{\partial}{\partial y}$	$\frac{\partial}{\partial z}$	$-j$	$\frac{\partial}{\partial x}$	$\frac{\partial}{\partial z}$	$+k$	$\frac{\partial}{\partial x}$	$\frac{\partial}{\partial y}$
	$-3xz$	$2xy$		yz	$2xy$		yz	$-3xz$	$-2xy$

$$\nabla \times B = i \left(\frac{\partial (2xy)}{\partial y} + \frac{\partial (3xz)}{\partial z} \right) - j \left(\frac{\partial (2xy)}{\partial x} - \frac{\partial (yz)}{\partial z} \right) + k \left(\frac{\partial (-3xz)}{\partial x} - \frac{\partial (yz)}{\partial y} \right)$$

$$\nabla \times B = i(2x + 3x) - j(2y - y) + k(-3z - z)$$

$$\nabla \times B = (5x)i - j(y) - k(4z)$$

$$\nabla \times B \Big|_{(1,2,1)} = 5(1)i - (2)j - 4(1)k$$

$$\nabla \times B \Big|_{(1,2,1)} = 5i - 2j - 4k$$

iv) $\nabla \text{grad div } A$

$$\text{div } A = \nabla \cdot A$$

and from the previously solved (ii)

$$\nabla \cdot A = 2xy + (x+z) + 2xz$$

$$\therefore \text{grad div } A = \nabla(\nabla \cdot A)$$

$$\text{grad div } A = \left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) (2xy + x + z + 2xz)$$

$$\text{grad div } A = i \frac{\partial}{\partial x} (2xy + x + z + 2xz) + j \frac{\partial}{\partial y} (2xy + x + z + 2xz)$$

$$+ k \frac{\partial}{\partial z} (2xy + x + z + 2xz)$$

$$\text{grad div } A = i(2y + 1 + 2z) + j(2x) + k(1 + 2x)$$

$$\text{grad div } A \Big|_{(1,2,1)} = (2(2) + 1 + 2(1))i + (2(1))j + (1 + 2(1))k$$

$$\text{grad div } A \Big|_{(1,2,1)} = 7i + 2j + 3k$$

v) curl A =

$$\text{curl } A = \nabla \times A$$

$$\nabla \times A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz^2 & (y+z) & (xz) \end{vmatrix}$$

$$\nabla \times A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz^2 & (y+z) & (xz) \end{vmatrix} = \hat{i} \left(\frac{\partial}{\partial y} (xz) - \frac{\partial}{\partial z} (y+z) \right) - \hat{j} \left(\frac{\partial}{\partial x} (xz) - \frac{\partial}{\partial z} (xz^2) \right) + \hat{k} \left(\frac{\partial}{\partial x} (y+z) - \frac{\partial}{\partial y} (xz^2) \right)$$

$$\nabla \times A = \hat{i} \left(\frac{\partial}{\partial y} (xz) - \frac{\partial}{\partial z} (y+z) \right) - \hat{j} \left(\frac{\partial}{\partial x} (xz) - \frac{\partial}{\partial z} (xz^2) \right) + \hat{k} \left(\frac{\partial}{\partial x} (y+z) - \frac{\partial}{\partial y} (xz^2) \right)$$

$$\nabla \times A = \hat{i} (0 - 1) - \hat{j} (z^2 - 0) + \hat{k} (1 + z - z^2)$$

$$\nabla \times A = (-1)\hat{i} - (z^2)\hat{j} + (1+z-z^2)\hat{k}$$

$$\text{curl } (\text{curl } A) = \nabla \times (\nabla \times A)$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -1 & -z^2 & (1+z-z^2) \end{vmatrix}$$

$$\nabla \times (\nabla \times A) = \hat{i} \left(\frac{\partial}{\partial y} (1+z-z^2) - \frac{\partial}{\partial z} (-z^2) \right) - \hat{j} \left(\frac{\partial}{\partial x} (1+z-z^2) - \frac{\partial}{\partial z} (-1) \right) + \hat{k} \left(\frac{\partial}{\partial x} (-z^2) - \frac{\partial}{\partial y} (-1) \right)$$

$$\nabla \times (\nabla \times A) = \hat{i} \left(\frac{\partial}{\partial y} (1+z-z^2) + \frac{\partial}{\partial z} (z^2) \right) - \hat{j} \left(\frac{\partial}{\partial x} (1+z-z^2) + \frac{\partial}{\partial z} (-1) \right) + \hat{k} \left(\frac{\partial}{\partial x} (-z^2) + \frac{\partial}{\partial y} (-1) \right)$$

$$\nabla \times (\nabla \times A) = \hat{i} (0 + 2z) - \hat{j} (-2z + 0) + \hat{k} (0 + 1)$$

$$\nabla \times (\nabla \times A) = (1+2z)i + (2z)i + k$$

$$\nabla \times (\nabla \times A) \Big|_{(1,2,1)} = (1+2(1))i + (2(1))i + k$$

$$\nabla \times (\nabla \times A) \Big|_{(1,2,1)} = 3i + 2j + k$$