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Question 1.

i). Mathematical Modelling process of setting up of the model, solving it mathematically and interpreting the results in physical or other terms.

ii. Methods of developing model in engineering System:

(a) Exponential Growth

(b) Radioactive Dating.

Question 2.

If $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t} k$.

determine (i) d/dt (ii) $\frac{d^2r}{dt^2}$ and (iii) the value of $\left|\frac{d^2r}{dt^2}\right|$ at $t = 0$.

Solution.

$r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t} k$

i. $\frac{dr}{dt} = (2t + 3)i - (6\cos 3t)j + (6e^{2t})k$

ii. $\frac{d^2r}{dt^2} = 2i + (18\sin 3t)j + (12e^{2t})k$.

iii. $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0)j + 12e^{2(0)}k$.

$= 2i + 12k$.

$\left|\frac{d^2r}{dt}\right|_{t=0} = \sqrt{2^2 + 12^2}$

$\sqrt{4 + 144}$

$\sqrt{148}$

$= 12.2$ units

Question 3

If $A = x^2 y i + (xy + yz) j + xz^2 k$

$B = yz i - 3xz j + 2xy k$ and

$\phi = 3x^2 y + xyz - 4y^2 z^2 - 8$

Determine at the point $(1, 2, 1)$

(i) $\nabla\phi$ (ii) $A \cdot B$, (iii) $\nabla \times B$ (iv) grad div A and (v) Curl Curl A.

Solution

$A = x^2 y i + (xy + yz) j + xz^2 k$.

$B = yz i - 3xz j + 2xy k$.

$\phi = 3x^2 y + xyz - 4y^2 z^2 - 3$.

i $\quad \nabla\phi = \left[i \frac{d}{dx} + j \frac{d}{dy} + k \frac{d}{dz} \right] \phi$.

$i \frac{d\phi}{dx} + j \frac{d\phi}{dy} + k \frac{d\phi}{dz}$

$\nabla \quad = i(yz + 6xy) + j(3x^2 + xz - 8yz^2) + k[xy$

$\nabla \times$ at point $(1, 2, 1)$

$\nabla\phi = i[2 \times 1 + 12] + j(3 \times 1^2 + 1 \times 1 - 8 \times 2 \times 1^2] + k[1 \times$

Grad $\nabla\phi = i(14) + j(-12) + k(-30)$.

$\nabla\phi = 14i - 12j + 30k$.

$$\nabla \cdot A = \frac{d}{dx} \cdot a_x + \frac{d}{dy} \cdot a_y + \frac{d}{dz} \cdot a_z$$

$$= 2xy + (x+z) + 2xz$$

$\nabla \cdot A =$ at $(1, 2, 1)$.

$$\nabla \cdot A = 2 \times 1 \times 2 + (1+1) + 2 \times 1 \times 1$$

$$\nabla \cdot A = 4 + 2 + 2$$

$$\nabla \cdot A = 8$$

iii. $A \times B = \begin{bmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{bmatrix}$

$$i \begin{vmatrix} d/dy & d/dz \\ -3xz & 2xy \end{vmatrix} - j \begin{vmatrix} d/dx & d/dz \\ yz & 2xy \end{vmatrix} + k \begin{vmatrix} \frac{d}{dx} & \frac{d}{dy} \\ yz & -3xz \end{vmatrix}$$

$$i \left(\frac{d}{dy}(2xy) - \frac{d}{dz}(-3xz) \right) - j \left(\frac{d}{dx}(2xy) - \frac{d}{dz}(yz) \right) +$$

$$= (2x + 3x)i - j(2y - y) + k(-3z - z).$$

$\nabla \times B$ at $(1, 2, 1)$.

$$\nabla \times B = (2 \times 1 + 3 \times 1)i - j(2 \times 2 - 2) + k(-3 \times 1 - 1)$$

$$\nabla \times B = 5i - 2j - 4k.$$

$\nabla(\nabla \cdot A)$ at $(2,2,1)$

$\nabla(\nabla \cdot A) = (2\times2+1+2\times1)i + (2\times1)j + (1+2\times1)k$

$= (7)i + (2)j + 3k$

$7i + 2j + 3k$

Curl Curl

Curl A $= \nabla \times A = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ (x^2y) & (xy+yz) & (xz^2) \end{vmatrix}$

$i \begin{vmatrix} d/dy & d/dz \\ (xy+yz) & (xz^2) \end{vmatrix} - j \begin{vmatrix} d/dx & d/dz \\ (x^2y) & (xz^2) \end{vmatrix} + k \begin{vmatrix} d/dx & d/ \\ x^2y & (\end{vmatrix}$

$i\left(d/dy(xz^2) - d/dz(xy+yz)\right) - j\left(d/dx(xz^2) - d/dz\, x^2y\right] + k \left| d/d \right.$

$i(0-y) - j(z^2-0) + k(y-x^2)$

$i(0-y) - j(z^2-0) + (y-x^2)k$

$\nabla \times A = -yi - z^2j + (y-x^2)k$

$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ -y & -z^2 & (y-x^2) \end{vmatrix}$