

Ebitur Paul Nsien

17/ENG06/095

Mechanical Engineering

i) Mathematical modelling is the process of developing a mathematical model using a description of a system using mathematical concepts and language.

It is also the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight answers and guidance useful for the originating application.

ii) Using law of conservation of mass

2 If  $r = (t^2 + 3t)i - 2\sin 3tj + 3e^{2t}k$

Determine

$$\frac{dr}{dt} = (2t + 3)i - 6\cos 3tj + 6e^{2t}k$$

Question (11)

$$\frac{d^2r}{dt^2} = 2i + 18\sin 3tj + 12e^{2t}k$$

$$= \sqrt{(2)^2 + [18\sin 3(0)]^2 + [12e^{2\cos}]^2}$$

$$= \sqrt{4 + 0 + (12)^2}$$

$$= \sqrt{148}$$

$$= 12.166$$

Question 3

i) If

$$A = x^2y i + (2y + yz)j + xz^2 k$$

$$B = yz i - xz j + 2xy k$$

$$Q = 3x^2y + 4yz - 4y^2z^2 - 3$$

Determine, at the point (1, 2, 1)

~~Problem 1~~

$$i) \quad \nabla \phi = \text{grad } \phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k$$

$$\frac{\partial \phi}{\partial x} = 6xy + yz$$

$$\frac{\partial \phi}{\partial y} = 3x^2 + xz - 8yz^2$$

$$\frac{\partial \phi}{\partial z} = xy - 8y^2z$$

$$\nabla \phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)k$$

at point (1, 2, 1)

$$\nabla \phi = [6(1)(2) + (2)(1)]i + [3(1)^2 + (1)(1) - 8(2)(1)^2]j + [(1)(2) - 8(2)^2(1)]k$$

$$= (12 + 2)i + (3 + 1 - 16)j + [2 - 32]k$$

$\nabla \cdot A$

$$\nabla = \left( \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right) (x^2 y i + (xy + yz)j + xz^2 k)$$

$$i \cdot i = j \cdot j = k \cdot k = 1$$

$$i \cdot j = j \cdot k = k \cdot i = 0$$

$$\nabla \cdot A = \frac{\partial}{\partial x} (x^2 y) i \cdot i + \frac{\partial}{\partial y} (xy + yz) j \cdot j + \frac{\partial}{\partial z} (xz^2) k \cdot k$$

$$= 2xy + (x + z) + 2xz$$

at point (1, 2, 1)

$$\nabla \cdot A = 2(1)(2) + [1 + 1] + 2(1)(1) = 4 + 2 + 2 = 8$$

ii)  $\nabla \times B$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -3xz & 2xy \end{vmatrix}$$

$$= \left[ \frac{\partial}{\partial y} (2xy) - \frac{\partial}{\partial z} (-3xz) \right] i - \left[ \frac{\partial}{\partial x} (2xy) - \frac{\partial}{\partial z} (yz) \right] j$$

$$+ \left[ \frac{\partial}{\partial x} (-3xz) - \frac{\partial}{\partial y} (yz) \right] k$$

$$= [2x + 3x] i - [2y - y] j + [-3z - z] k$$

$$\nabla \times B = 5xi - yj - 4zk$$

at point  $(1, 2, 1)$

$$\nabla \times B = 5(1)i - (2)j - 4(1)k$$

$$= 5i - 2j - 4k = (5, -2, -4)$$

iv) grad div A

$$A = x^2 y i + (xy + yz) j + xz^2 k$$

$$\text{div } A = \frac{\partial}{\partial x} (x^2 y) + \frac{\partial}{\partial y} (xy + yz) + \frac{\partial}{\partial z} (xz^2)$$

$$\text{div } A = 2xy + (x + z) + 2xz$$

$$\text{grad div } A = \text{grad } Q = \nabla Q$$

$$\nabla Q = \frac{\partial Q}{\partial x} i + \frac{\partial Q}{\partial y} j + \frac{\partial Q}{\partial z} k$$

$$\nabla Q = (2y + 1 + 2z) i + (x + z) j + (1 + 2x) k$$

at point  $(1, 2, 1)$

$$\nabla Q = (2(1) + 1 + 2(1)) i + (1 + 1) j + (1 + 2(1)) k$$

$$\nabla Q = 5i + 2j + 3k$$

$$\text{grad div } A = 5i + 2j + 3k = (5, 2, 3)$$

v)  $A = x^2 y i + (xy + yz) j + xz^2 k$

$$\text{curl } A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 y & (xy + yz) & xz^2 \end{vmatrix}$$

$$i \left[ \frac{d}{dy}(xz^2) - \frac{d}{dz}(xy+yz) \right] - j \left[ \frac{d}{dx}(xz^2) - \frac{d}{dz}(x^2y) \right]$$

$$+ k \left[ \frac{d}{dx}(xy+yz) - \frac{d}{dy}(x^2y) \right]$$

$$\text{curl } A = [0-y]i - [z^2-0]j + [y-x^2]k = (-y, z^2, y-x^2)$$

$$\text{at } (1, 2, 1) \Rightarrow -2i - (1^2)j + (2-1^2)k$$

$$\text{curl } A = -2i - j + k = (-2, -1, 1)$$

$$(-2, -1, 1) //$$