

NWOSU IKEDINACHI . U.

16/ ENGO61043

MECHANICAL

2)  $\nabla \phi$  at point  $(1, 2, 1)$

$$\nabla \phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k$$

$$\phi = 6xy + yz$$

$$\frac{\partial \phi}{\partial x} = 3x^2 + xz - 8yz^2$$

$$\frac{\partial \phi}{\partial y} = 2y - 8y^2z$$

$$\nabla \phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (2y - 8y^2z)k$$

at point  $(1, 2, 1)$

$$\nabla \phi = (12+2)i + (3-8(2)(2))j + (2-32)k$$

$$= 14i + (4-16)j + (-30)k$$

$$= 14i - 12j - 30k$$

$$\nabla \phi = 14i - 12j - 30k$$

3)  $\nabla \cdot A$

$$\nabla \cdot A = \frac{d}{dx} i + \frac{d}{dy} j + \frac{d}{dz} k \cdot (k^2y + (6xy + yz)i + xz^2k)$$

$$\nabla \cdot A = \frac{d}{dx} (xz^2) + \frac{d}{dy} (6xy + yz) + \frac{d}{dz} (k^2y)$$

$$\nabla \cdot A = 2xz + (x+z) + 0 + (2xz)$$

$$= 2(1)(2) + (1+1) + (2 \times 1 \times 1) = 4 + 2 + 2$$

$$\nabla \cdot A = 8$$

1)  $\nabla \times B =$

i	j	k
$\frac{d}{dx}$	$\frac{d}{dy}$	$\frac{d}{dz}$
$yz$	$-3xz$	$2xy$

$$i \left( \frac{d}{dy} (2xy) + \frac{d}{dz} (3xz) \right) - j \left( \frac{d}{dx} (2xy) - \frac{d}{dz} (yz) \right)$$

$$+ k \left( \frac{d}{dz} (-3xz) - \frac{d}{dy} (yz) \right)$$

$$i(2x+2z) - j(2y-y) + k(-3z-z)$$

$$i(5x) - j(y) + k(-4z)$$

$$5xi - yj - 4zk$$

at point (1, 2, 1)

$$\nabla \times A = 5i - 2j - 4k$$

ii) grad div A  $\nabla \cdot A$

$$\text{Div } A = 2xy + (x+z) + 2xz$$

$$\nabla(\nabla \cdot A) = \frac{d(\nabla \cdot A)}{dx} i + \frac{d(\nabla \cdot A)}{dy} j + \frac{d(\nabla \cdot A)}{dz} k$$

$$\begin{aligned} \nabla(\nabla \cdot A) &= i(2y+1+2z) + 2xj + (1+2x)k \\ &= i(4+1+2) + 2j + 3k \\ &= 7i + 2j + 3k \end{aligned}$$

iii)  $\nabla \times (\nabla \times A)$   $A = x^2y i + (xy + yz) j + xz^2 k$

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$i(0-y) - j(z^2-0) + k(x^2+y) - yj - j(z^2) + k(x^2+y)$$

$$\nabla \times A = -y i - j(z^2) + k(x^2+y)$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -y & -z^2 & x^2+y \end{vmatrix}$$

$$= i(1+z^2) - j(0-2x) + k(1-0)$$

$$= (1+z^2)i + 2xj + k$$

$$= (1+2(1))i + 2j + k$$

$$\nabla \times (\nabla \times A) = 3i + 2j + k$$



## Solution

1) Mathematical modelling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and elemental analysis provides insight, answers, and guidance useful for the originating application.

b) In chemical engineering: chemical equilibrium

i) In electrical engineering: Power supply network optimization

$$2) \quad r = (t^2 + 3t)i - 25\sin 3tj + 3e^{2t}k$$
$$\frac{dr}{dt} = (2t + 3)i - 6\cos 3tj + 6e^{2t}k$$

$$\frac{d^2r}{dt^2} = 2i + 18\sin 3tj + 12e^{2t}k$$

$$\left| \frac{d^2r}{dt^2} \right| \text{ at } t=0$$

$$\Rightarrow 2i + 18\sin(3 \times 0)j + 12e^{2(0)}k$$
$$= 2i + 12k$$

$$\left| \frac{d^2r}{dt^2} \right| = \sqrt{4 + 144} = \sqrt{148} = 2\sqrt{37}$$

$$= 12.6556$$

$$3) \quad A = x^2y i + (xy + yz)j + xz^2 k$$
$$B = yz i - 3xz j + 2xy k$$
$$D = 3x^2y + xyz - 4y^2z^2 - 3$$