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16/ENG04/045
ELECT/ELECT

Solution

1) Mathematical modelling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers, and guidance useful for the originating application

b) a) In chemical engineering: chemical equilibrium
b) In electrical engineering: Power supply network optimization

2) $r = (t^2+3t)i - 2\sin 3t j + 3e^{2t}k$

$\frac{dr}{dt} = (2t+3)i - 6\cos 3t j + 6e^{2t}k$

$\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t}k$

$\left|\frac{d^2r}{dt^2}\right|$ at $t=0$

$\Rightarrow 2i + 18\sin(3\times 0)j + 13e^{2(0)}k$

$= 2i + 12k$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{4+144} = \sqrt{148} = 2\sqrt{37}$

$= 12.1655$

3) $A = x^2yi + (xy+yz)j + xz^2k$

$B = yzi - 3xzj + 2xyk$

$\emptyset = 3x^2y + xyz - 4y^2z^2 - 3$

i) $\nabla\phi$ at point $(1,2,1)$

$$\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{d\phi}{dz}k$$

$$\frac{d\phi}{dx} = 6xy + yz$$

$$\frac{d\phi}{dy} = 3x^2 + xz - 8yz^2$$

$$\frac{d\phi}{dz} = xy - 8y^2z$$

$$\nabla\phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)k$$

at point $(1,2,1)$

$$\nabla\phi = (12+2)i + (3+1-8(2)(2))j + (2-32)k$$
$$= 14i + (4-16)j + (-30)k$$
$$= 14i - 12j - 30k$$
$$\nabla\phi = 14i - 12j - 30k$$

ii) ∇A

$$\nabla \cdot A = \frac{d}{dx}i + \frac{d}{dy}j + \frac{d}{dz}\hat{k} \cdot (x^2y i + (xy+yz)j + xz^2 k)$$

$$\nabla \cdot A = \frac{d}{dx}(x^2y) + \frac{d}{dy}(xy+yz) + \frac{d}{dz}(xz^2)$$

$$\nabla \cdot A = 2xy + (x+z) + 0 + (2xz)$$
$$= 2(1)(2) + (1+1) + (2\times1\times1) = 4+2+2$$
$$\nabla \cdot A = 8$$

iii) $\nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$

$$i\left(\frac{d}{dy}(2xy) + \frac{d}{dz}(3xz)\right) - j\left(\frac{d}{dx}(2xy) - \frac{d}{dz}(yz)\right)$$
$$k\left(\frac{d}{dz}(-3xz) - \frac{d}{dy}(yz)\right)$$

$i(2x+3x) - j(2y-y) + k(-3z-z)$

$i(5x) - j(y) + k(-4z)$

$5xi - yj - 4zk$

at point $(1,2,1)$

$\nabla \times B = 5i - 2j - 4k$

ii) grad div A $\quad \nabla \cdot \vec{A}$

$\text{Div } A = 2xy + (x+z) + 2zx$

$$\nabla(\nabla \cdot A) = \frac{d(\nabla \cdot A)}{dx} i + \frac{d(\nabla \cdot A)}{dy} j + \frac{d(\nabla \cdot A)}{dz} k$$

$\nabla(\nabla \cdot A) = i(2y+1+2z) + 2xj + (1+2x)k$

$= i(4+1+2) + 2j + 3k$

$= 7i + 2j + 3k$

v) $\nabla \times (\nabla \times \vec{A})$ $\quad \vec{A} = x^2 y i + (xy + yz) j + xz^2 k$

$$\nabla \times \vec{A} = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ x^2 y & (xy+yz) & (xz^2) \end{vmatrix}$$

$i(0-y) - j(z^2 - 0) + k(x^2 + (y)) - yi - j(z^2) + k(x^2 - y)$

$\nabla \times \vec{A} = -yi - j(z^2) + k(x^2 - y)$

$$\nabla \times (\nabla \times \vec{A}) = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ -y & -z^2 & x^2 - y \end{vmatrix}$$

$= i(1+2z) - j(0-2x) + k(1-0)$

$= (1+2z)i + 2xj + k$

$= (1+2(1))i + 2j + k$

$\nabla \times (\nabla \times \vec{A}) = 3i + 2j + k$

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