AHLOMERETSE EDDRITSE N TEMERE
MECHATRONICS ENGINEERING
16/ENG05/014

1 Mathematical modelling is a process of setting up a model, solving it mathematically and interpreting the result in physical or other terms.

ii Numeric method by Euler

b Extended method by reduction to seperable form

2 $r=(t^2+3t)i - 2\sin 3t\, j + 3e^{2t} k$

i $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t} k$

ii $\frac{d^2r}{dt} = 2i + 18\sin 3t\, j + 12e^{2t} k$

iii $\left|\frac{d^2r}{dt^2}\right|_{at\ t=0} = 2i + 18\sin 3(0)\, j + 12e^{2(0)} k$

$= 2i + 12k$

$= \sqrt{2^2+12^2}$

$= \sqrt{4+144}$

$\left|\frac{d^2r}{dt^2}\right|_{at\ t=0} = 12.17$

3 $A = x^2yi + (xy+yz)j + xz^2 k$

$B = yzi - 3xzj + 2xyk$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

Points $(1,2,1)$

i $\nabla\phi = \left[i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right]\phi$

$= i\frac{d\phi}{dx} + j\frac{d\phi}{dy} + k\frac{d\phi}{dz}$

$= (6xy+yz)i + (3x^2+yz-8yz^2)j + (xy-8y^2z)k$

at point $(1,2,1)$

$\nabla\phi = (6(1)(2)+(2)(1))i + (3(1)^2+(2)(1)-8(2)(1)^2)j + ((1)(2)-8(2)^2(1))k$

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$\nabla\phi = 14i - 12j - 30k$

ii $\nabla \cdot A = \left[i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right] \cdot [axi + ayj + azk$

$= \frac{d}{dx} \cdot ax + \frac{d}{dy} \cdot ay + \frac{d}{dx} az$

$= 2xy + (x+z) + 2xz$

at points $(1, 2, 1)$

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$= 4 + 2 + 2$

$\nabla \cdot A = 8$

iii $\nabla \times B = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$

$= i\begin{vmatrix} d/dy & d/dz \\ -3xz & 2xy \end{vmatrix} - j\begin{vmatrix} d/dx & d/dz \\ yz & 2xy \end{vmatrix} + k\begin{vmatrix} d/dx & d/dy \\ yz & -3xz \end{vmatrix}$

$= i\left(\frac{d}{dy}(2xy) - \frac{d}{dz}(-3xz)\right) - j\left(\frac{d}{dx}(2xy) - \frac{d}{dz}(yz)\right) + k\left(\frac{d}{dx}(-3xz) - \frac{d}{dy}(yz)\right)$

$= (2x + 3x)i - (2y - y)j + (-3x - z)k$

at points $(1, 2, 1)$

$\nabla \times B = (2(1) + 3(1))i - (2(2) - 2)j + (-3(1) - 1)k$

$= 5i - 2j - 4k$

iv grad div A

div A, $\nabla \cdot A = 2xy + (x+z) + 2xz$

$\therefore$ grad div $A = \left(i\frac{d}{dx} + j\frac{d}{dy} + k\frac{d}{dz}\right) \cdot \nabla \cdot A$

$= i\frac{d\nabla \cdot A}{dx} + j\frac{d\nabla \cdot A}{dy} + k\frac{d\nabla \cdot A}{dz}$

$= (2y + z + 2z)i + 2xj + (x + 2x)k$

at points $(1, 2, 1)$

grad div $A = (2(2) + 1 + 2(1))i + 2(1)j + (1 + 2(1))k$

grad div $A = 7i + 2j + 3k$

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curl curl A

$$\text{curl } A = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= i\left|\frac{d}{dy}(xz^2) - \frac{d}{dz}(xy+yz)\right| - j\left|\frac{d}{dx}(xz^2) - \frac{d}{dz}(x^2y)\right| + k\left|\frac{d}{dx}(xy+yz) - \frac{d}{dy}(x^2y)\right|$$

$$= i(0-(0+y)) - j(z^2-0) + k((y+0) - x^2)$$

$$\text{curl } A = -yi - z^2j + (y - x^2k)$$

attempt

$$\text{curl curl } A = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= i\left|\frac{d}{dy}(y-x^2) - \frac{d}{dz}(-z^2)\right| - j\left|\frac{d}{dx}(y-x^2) - \frac{d}{dz}(-y)\right| + k\left|\frac{d}{dx}(-z^2) - \frac{d}{dy}(-y)\right|$$

$$= i((1-0)+2z) - j((0-2x)-0) + k(0+1)$$

$$= (1+2z)i + 2xj + k$$

at point (1, 2, 1)

$$\text{curl curl } A = (1+2(1))i + 2(1)j + k$$

$$= 3i + 2j + k$$

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