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C ENG282
D Elect/Elect
M 16/ENG04/024

Question and Answers

1) A mathematical model a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical and other terms.

2) i) Exponential Growth Decay (use of ODE)
ii) Mixing problems

$r = (t^2 + 3t)i - 2\sin 3t\, j + 3e^{2t} k$

a) $dr/dt = (2t+3)i - 6\cos 3t\, j + 6e^{2t} k$

ii) $d^2r/dt^2 = 2i + 18\sin 3t\, j + 12e^{2t} k$

iii) $\frac{d^2r}{dt^2} = 2i + 12k$

$\left| d^2r/dt^2 \right| = \sqrt{2^2 + 12^2} = \sqrt{4 + 144} = \sqrt{148} = 2\sqrt{37} = 12.17$

3) $A = x^2yi + (xy + y^2)j + xz^2k$

$B = yzi - 3xzj + 2xyk$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

i) $\nabla\phi = \frac{d\phi}{dx} i + \frac{d\phi}{dy} j + \frac{d\phi}{dz} k$

$$\frac{d\phi}{dx} = 6xy + yz$$

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$\nabla(\nabla A) = \nabla c = i\frac{\partial c}{\partial x} + j\frac{\partial c}{\partial y} + k\frac{\partial c}{\partial z}$

$= i(2y + 1 + 2z) + j(2x) + k(1 + 2x)$

At $(1, 2, 1)$

$\nabla c = i[2(2) + 1 + 2(1)] + j[2(1)] + k[1 + (2)(1)]$

$= i(4 + 1 + 2) + j(2) + k(1 + 2)$

$= 7i + 2j + 3k$

v Curl Gm A

$\text{Curl } A = \nabla \times A$

$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & (xy + yz) & xz^2 \end{vmatrix}$

$= i(0 - y) - j(z^2 - 0) + k(y - 0x^2)$

$= -yi - z^2j + k(y - x^2)$

At $(1, 2, 1)$

$\text{Curl } A = -2i - j + k$

$\text{Curl } A = \nabla \times (\nabla \times A)$

$\nabla \times (\nabla \times A)$

$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & -z^2 & (y - x^2) \end{vmatrix}$

$= i(1 + 2z) - j(-2x - 0) + k(0 + 1)$

$= i(1 + 2z) + 2x^2j + k$

At point $(1, 2, 1)$

$\nabla \times (\nabla \times A) = i(1 + 2(1)) + 2(1)^2j + k$

$= 3i + 2j + k$

$\frac{\partial \phi}{\partial y} = 3x^2 + xz - 8yz^2$

At (1, 2, 1)

$\frac{\partial \phi}{\partial x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$

$\frac{\partial \phi}{\partial y} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3 + 1 - 16$

$= -12$

$\frac{\partial \phi}{\partial z} = (1)(2) - 8(2)^2(1) = 2 - 32$

$= -30$

$\therefore \nabla \phi = 14i - 12j - 30k$

ii) $\nabla \cdot A = \frac{\partial a}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$A = axi + ayj + azk$

$\nabla \cdot A = 2xy + (x + z) + 2xz$

At (1, 2, 1)

$\nabla \cdot A = 2(1)(2) + (1 + 1) + 2(1)(1)$

$= 4 + 2 + 2 = 8$

iii) $\nabla \times B = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$

$= i(2x + 3x) - j(2y - y) + k(-3z - z)$

$= 5xi - yj - 4zk$

At (1, 2, 1)

$\nabla \times B = 5i - 2j - 4k$

iv) grad div A

grad $(2x + (x + z) + 2xz)$