Name : Kemebigha favour Denowei
Matric No : 17/GNG04/085
Department : Elect /Elect

GNG 252 ASSIGNMENT

1)i) Mathematical modelling is defined as a process of developing a mathematical model

ii) Methods of developing model in engineering system are:
a) Radioactivity
b) Mixing problems

2

i.e $r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t}k$

i) $\frac{dr}{dt} = (2t+3)i - 2(3\cos 3t)\,j + 2(3e^{2t})k$

$= (2t+3)i - 6\cos 3t\, j + 6e^{2t}k$

ii) $\frac{d^2r}{dt^2} = \frac{d(2t+3)}{dt}i - \frac{d(6\cos 3t)}{dt}j + \frac{d}{dt}(6e^{2t})k$

$= 2i - (-(6\times 3)(\sin 3t)\,j) + 12e^{2t}k$

$= 2i + 18\sin 3t\, j + 12e^{2t}k.$

Value of $|d^2r/dt^2|$ at $t = 0$

$= 2i + 18\sin 3(0)\,j + 12e^{2(0)}k$

$= 2i + 0j + 12k$

$= \sqrt{2^2+12^2} = \sqrt{4+144}$

$= 2\sqrt{37}$ units.

3

i) $\nabla\phi$

When $\phi = 3x^2y + xyz - 4y^2z^2 - 3$

$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} - k\frac{\partial\phi}{\partial z}$

$\frac{\partial\phi}{\partial x} = 6xy + yz - 0$

$\frac{\partial\phi}{\partial y} = 3x^2 + xz - 8yz^2$

$\frac{\partial\phi}{\partial z} = xy - 8y^2z$

$\nabla\phi = (6xy + yz)i + (3x^2 + xz - 8y^2z^2)j + (xy - 8y^3z)k$

at point (1, 2, 1)

$x = 1$, $y = 2$ and $z = 1$

$= i((6(1)(2)) + ((2)(1)) + j(3(1)^2 + (1)(1) - 8(2)(1)^2)j + ((1)(2) - 8(2)^2(1))k$

$= (12 + 2)i - 12j - 30k$

$\nabla\phi = 14i - 12j - 30k$

ii) $\nabla \cdot A$

$= \frac{\partial Ax}{\partial x} + \frac{\partial Ay}{\partial y} + \frac{\partial Az}{\partial z}$

$\nabla \cdot A = 2xy + (x + z) + 2xz$

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$= (2 \times 2) + 2 + 2$

$= 4 + 2 + 2$

$= 8$

iii) Curl of $B = \nabla \times B$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3x & 2xy \end{vmatrix}$$

$\nabla \times B = i\left(\frac{\partial 2xy}{\partial y} + \frac{\partial 3xz}{\partial z}\right) - j\left(\frac{\partial 2xy}{\partial x} - \frac{\partial yz}{\partial z}\right) + k\left(-\frac{\partial 3xz}{\partial x} - \frac{\partial yz}{\partial y}\right)$

$= (2x + 3x)i - (2y - y)j + (-3z - z)k$

$= 5xi - yj - 4zk$

at (1, 2, 1)

$= (2(1) + 3(1))i - (2(2) - 2)j + (-3(1) - 1)k$

$\nabla \times B = 5i - 2j - 4k$

iv)

iv) gradient of div $A = \nabla(\nabla \cdot A)$

$\nabla \cdot A = 2xy + (x+z) + 2xz$

$\nabla(\nabla \cdot A) = (2y + 2 + 2z)i + 2x j + (x + 2x)k$

$\nabla(\nabla \cdot A)$ at $(1,2,1)$

$\nabla(\nabla \cdot A) = (2(2) + 1 + 2(1))i + 2(1)j + (1 + 2(1))k$

$= (4 + 1 + 2)i + 2j + 3k$

$= 7i + 2j + 3k$

v) Curl curl A

$$\text{curl } A = \nabla \times A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & (xz) \end{vmatrix}$$

$$= i \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy+yz) & (xz^2) \end{vmatrix} - j \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2y & (xz^2) \end{vmatrix} + k \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2y & (xy+yz) \end{vmatrix}$$

$= i\left(\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy+y^2)\right) - j\left(\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right) + k\left[\frac{\partial}{\partial y}(xy+y^2) - \frac{\partial}{\partial y}(x^2y)\right]$

$= i(0-y) - j(z^2-0) + k(y-x^2)$

$= -yi - z^2j + (y-x^2)k$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$= i\left[\frac{\partial}{\partial y}(y-x^2) - \frac{\partial}{\partial z}(-z)^2\right] - j\left[\frac{\partial}{\partial x}(y-x^2) - \frac{\partial}{\partial z}(-y)\right] + k\left[\frac{\partial}{\partial x}(-z^2) - \frac{\partial}{\partial y}(-y)\right]$

$= (1+2z)i - j(2x+1) + k(0+1)$

at $(1,2,1)$

$\nabla \times (\nabla \times A) = i(1+2(1)) - j(-2\times1) + k(1)$

$= 3i + 2j + k.$