

9/11/23

1) Mathematical modelling is a way of representing a problem for an optimization with quantitative mathematical relationships which describe and model analysis, predict, design, control, and optimize systems. It is a key to engineering optimization.

2) Basic differential programming: classical optimization
 a) Linear differential programming: linear supply/rebate optimization

2) $f = (x^2 + 3x + 1) - 25 \sin^2 y + 30 e^{2x} z$
 $f_x = (2x + 3) - 6 \cos^2 y + 60 e^{2x} z$

$\frac{df}{dx} = 2x + 18 \sin^2 y + 120 e^{2x} z$

$\left[\frac{df}{dx} \right]_{at \ x=0}$

$= 5 \quad 21 + 18 \sin^2(2 \times 0) + 120 e^{0 \times 2} z$

$\left| \frac{df}{dx} \right| = \sqrt{4 + 12z} = \sqrt{14z} = 0.589$
 $= 12.1655$

3) $A = x^2 y^2 + (x^2 y + y^2 z) + y z^2 x$
 $B = 3x^2 y + 3x^2 z + 2x^2 y z$
 $C = 3x^2 y + x y z - 4x^2 z^2 - 3$

1) $\nabla G = \nabla f(x, y, z)$
 $\nabla G = 0$

$$1) \nabla \phi \text{ at point } (1, 2, 1) \\ \nabla \phi = 2\phi \frac{\partial}{\partial x} + 2\phi \frac{\partial}{\partial y} + \frac{d\phi}{dz} k$$

$$\frac{d\phi}{dx} = 6xy + yz$$

$$\frac{d\phi}{dy} = 3x^2 + xz - 8yz^2$$

$$\frac{d\phi}{dz} = xy - 8yz$$

$$\nabla \phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8yz^2)k$$

at point (1, 2, 1)

$$\nabla \phi = (6+2)i + (3+2-8)j + (2-30)k \\ = 14i + (-4-16)j + (-30)k \\ = 14i - 12j - 30k$$

$$\nabla \phi = 14i - 12j - 30k$$

ii) ∇A

$$\nabla A = \frac{d}{dx} i + \frac{d}{dy} j + \frac{d}{dz} k \cdot (k^2y + (xy+yz)) + xz^2k$$

$$\nabla A = \frac{d}{dx} (x^2y) + \frac{d}{dy} (xy+yz) + \frac{d}{dz} (xz^2)$$

$$\nabla A = 2xy + (x+z) + 0 + (2xz) \\ = 2(1)(2) + (1+1) + (2 \times 1 \times 1) = 4+2+2$$

$$\nabla A = 8k$$

$$ii) \nabla \times B = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz - 3xz & 2xy & 2xy \end{vmatrix}$$

$$= i \left(\frac{\partial}{\partial y} (2xy) + \frac{\partial}{\partial z} (3xz) \right) - j \left(\frac{\partial}{\partial x} (2xy) - \frac{\partial}{\partial z} (yz) \right) \\ + k \left(\frac{\partial}{\partial x} (-3xz) - \frac{\partial}{\partial y} (2xy) \right)$$

$$\nabla f(x, y, z) = (2x-4, 2y-4, 2z-4)$$

at point (1, 2, 1)

$$\nabla f = (5, -2, -2)$$

ii) grad $\ln A = \nabla \cdot A$

$$\nabla(\nabla \cdot A) = \frac{d}{dx}(2x+2) + \frac{d}{dy}(2x) + \frac{d}{dz}(1+2x)k$$

$$= (2, 2, 2)k$$

iii) $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$

$$= (2, 2, 2)k - (2, 2, 2)k = 0$$

iv) $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$

$$= (2, 2, 2)k - (2, 2, 2)k = 0$$