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Electrical Electronics Engineering

Question and Answers

1) A Mathematical model a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical and other terms.

2) 1) Exponential growth Decay (use of ODE)
 ii) Many problems

$$r = (t^2 + 3t) \mathbf{i} - 2 \sin 3t \mathbf{j} + 3e^{2t} \mathbf{k}$$

a) $\frac{dr}{dt} = (2t + 3) \mathbf{i} - 6 \cos 3t \mathbf{j} + 6e^{2t} \mathbf{k}$

ii) $\frac{d^2 r}{dt^2} = 2 \mathbf{i} + 18 \sin 3t \mathbf{j} + 12e^{2t} \mathbf{k}$

iii) $\frac{d^2 v}{dt^2} = 2 \mathbf{i} + 12 \mathbf{k}$

$$\left| \frac{d^2 v}{dt^2} \right| = \sqrt{2^2 + 12^2} = \sqrt{4 + 144} = \sqrt{148} = 2\sqrt{37} = 12.17$$

3) $A = x^2 y \mathbf{i} + (xy + y^2) \mathbf{j} + xz^2 \mathbf{k}$

$$B = yz \mathbf{i} - 3xz \mathbf{j} + 2xy \mathbf{k}$$

$$\phi = 3x^2 y + xy^2 - 4y^2 z^2 - 3$$

i) $\nabla \phi = \frac{\partial \phi}{\partial x} \mathbf{i} + \frac{\partial \phi}{\partial y} \mathbf{j} + \frac{\partial \phi}{\partial z} \mathbf{k}$

$$\frac{\partial \phi}{\partial x} = 6xy + y^2$$

$$\frac{d\phi}{dy} = 3x^2 + xz - 8yz^2$$

$$\text{At } (1, 2, 1)$$

$$\frac{d\phi}{dx} = 6(1)(2) + (2)(1) = 12 + 2 = 14$$

$$\frac{d\phi}{dy} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3 + 1 - 16 = -12$$

$$\frac{d\phi}{dz} = (1)(2) - 8(2)^2(1) = 2 - 32 = -30$$

$$\Delta \nabla \phi = 14i - 12j - 30k$$

$$\text{ii) } \nabla \cdot A = \frac{da}{dx} + \frac{da_y}{dy} + \frac{da_z}{dz}$$

$$A = 2xz i + ay j + az k$$

$$\nabla \cdot A = 2xz + (x+2) + 2xz$$

$$\text{At } (1, 2, 1)$$

$$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$$

$$= 4 + 2 + 2 = 8$$

$$\text{iii) } \nabla \times B = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ yz & 3xz & 2xy \end{vmatrix}$$

$$= i(2xz + 3xz) - j(2y - y) + k(3z - z)$$

$$= 5xz i - y j - 4z k$$

$$\text{At } (1, 2, 1)$$

$$\nabla \times B = 5i - 2j - 4k$$

$$\text{iv) } \text{grad div } A$$

$$\text{and } (2xz + (x+2) + 2xz)$$

$$\nabla(\nabla A) = \nabla c = i \frac{dc}{dx} + j \frac{dc}{dy} + k \frac{dc}{dz}$$

$$= i(2y + 1 + 2z) + j(2x) + k(1 + 2z)$$

At (1, 2, 1)

$$\nabla c = i [2(2) + 1 + 2(1)] + j [2(1)] + k [1 + 2(1)]$$

$$= i(4 + 1 + 2) + j(2) + k(1 + 2)$$

$$= 7i + 2j + 3k$$

v. Curl of A

$$\text{Curl } A = \nabla \times A$$

$$= \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x^2y & (xy + yz) & xz^2 \end{vmatrix}$$

$$= i(0 - y) - j(x^2 - 0) + k(y - xz^2)$$

$$= -yi - x^2j + k(y - xz^2)$$

At (1, 2, 1)

$$\text{Curl } A = -2i - j + k$$

$$\text{Curl } A = \nabla \times (\nabla \times A)$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -y & -x^2 & (y - xz^2) \end{vmatrix}$$

$$= i(1 + 2z) - j(-2x - 0) + k(0 + 1)$$

$$= i(1 + 2z) + 2xj + k$$

At point (1, 2, 1)

$$\nabla \times (\nabla \times A) = i(1 + 2(1)) + 2(1)j + k$$

$$= 3i + 2j + k$$