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16/ENG01/009
CHEMICAL ENGINEERING.
ASSIGNMENT 3

1 A mathematical model is a description of a system using mathematical concepts and language. Modelling is the process of setting up a model solving it mathematically and interpreting the result in physical and other terms

bi Exponential growth / Decay (use of ODE)
ii mixing problems

2 $r = (t^2 + 3t)i - 2\sin 3t\, j + 3e^{2r} k$

i) $\frac{dr}{dt} = (2t + 3)\hat{i} - 6\cos 3t\,\hat{j} + 6e^{2r}\hat{k}$

ii) $\frac{d^2r}{dt^2} = 2\hat{i} + 18\sin 3t\,\hat{j} + 12e^{2r}\hat{k}$

iii) $\left.\frac{d^2r}{dt^2}\right|_{r=0} = 2\hat{i} + 12\hat{k}$

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$A = x^2 y i + (xy + yz) j + xz^2 k$

$B = yz i - 3xz j + 2xy k$

$\phi = 3x^2 y + xyz - 4y^2 z^2 - 3$

$$\nabla \phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} \hat{k}$$

$$\frac{d\phi}{dx} = 6xy + yz$$

$$\frac{d\phi}{dy} = 3x^2 + xz - 8yz^2$$

$$\frac{d\phi}{dz} = xy - 8y^2 z$$

$A \to (1, 2, 1)$

$$\frac{\partial \phi}{\partial x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$$

$$\frac{\partial \phi}{\partial y} = 3(1)^2 + (1)(1) - 8(2(1)^2 = 3 + 1 - 16 = -12$$

$$\frac{\partial \phi}{\partial z} = (1)(2) - 8(2)^2(1) = 2 - 32 = -30$$

$\partial x \quad \partial y \quad \partial z$

$A = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

$\nabla \cdot A = 2xy + (x+z) + 2xz$

$A + (1,2,1)$

$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$= 4 + 2 + 2 = 8$

$\nabla \times B$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$= \hat{i}\,(2x + 3x) - \hat{j}\,(2y - y) + \hat{k}(-3z - z)$

$= 5x\hat{i} - y\hat{j} - 4z\hat{k}$

$A + (1, 2, 1)$

$\nabla \times B = 5\hat{i} - 2\hat{j} - 4\hat{k}$

iv grad div A

grad $(2xy + (x+z) + 2xz)$

let Div A = C = ∇A

$\nabla(\nabla A) = \nabla C = \hat{i}\frac{\partial c}{\partial x} + \hat{j}\frac{\partial c}{\partial y} + \hat{k}\frac{\partial c}{\partial z}$

$= \hat{i}(2y + 1 + 2z) + \hat{j}(2x) + \hat{k}(1+$

At $(1, 2, 1)$

$\nabla C = \hat{i}(2(2) + 1 + 2(1)) + \hat{j}(2(1)) + \hat{k}(1 + 2(1))$

$= \hat{i}(4 + 1 + 2) + \hat{j}(2) + \hat{k}(1+2)$

$= 7\hat{i} + 2\hat{j} + 3\hat{k}$

v Curl A = $\nabla \times A$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & (xy + yz) & xz^2 \end{vmatrix}$$

$= \hat{i}(0 - y) - \hat{j}(z^2 - 0) + \hat{k}(y - x^2)$

$= -y\hat{i} - z^2\hat{j} + \hat{k}(y - x^2)$

At $(1, 2, 1)$

Curl A = $-2\hat{i} - \hat{j} + \hat{k}$

Curl curl A = $\nabla \times (\nabla \times A)$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= \hat{i}\,(1+2z) - \hat{j}\,(-2x^2 - 0) + \hat{k}\,(0+1)$$

$$= \hat{i}\,(1+2z) + 2x^2\,\hat{j} + \hat{k}$$

At $(1, 2, 1)$

$$\nabla \times (\nabla \times A) = \hat{i}\,(1+2(1)) + 2(1)^2\,\hat{j} + \hat{k}$$

$$= 3\hat{i} + 2\hat{j} + \hat{k}$$

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