PAM-DUSU M. TRUMAN
MECHANICAL ENGINEERING
16/ENG06/067
ENG 282
~~Mechanical Engineering~~

i Mathematical modelling is the process of developing a mathematical model using a discription of a system using mathematical concepts and language.
It is also the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight answers and guidance useful for the originating application.

ii Using law of conservation of mass

2 If $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t} k$

Determine

$$\frac{dr}{dt} = (2t+3)i - 6\cos 3t j + 6e^{2t} k$$

Question (11b)

$$\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t} k$$

$$= \sqrt{(2)^2 + [18\sin 3(0)]^2 + [12e^{2\cos}]^2}$$

$$= \sqrt{4 + (0)^2 + (12)^2}$$

$$= \sqrt{148}$$

$$= 12.166$$

Question 3

i If

$A = x^2yi + (2y + yz)j + xz^2k$

$B = yzi - xzj + 2xyk$

$Q = 3x^2y + xyz - 4y^2z^2 - 3$

Determine, at the point $(1, 2, 1)$

~~[illegible]~~

(1) $\nabla\phi = \text{grad } \phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$

$\frac{\partial\phi}{\partial x} = 6xy + yz$

$\frac{\partial\phi}{\partial y} = 3x^2 + xz - 8yz^2$

$\frac{\partial\phi}{\partial z} = xy - 8y^2z$

$\nabla\phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)k$

at point $(1,2,1)$

$\nabla\phi = [6(1)(2) + (2)(1)]i + [3(1)^2 + (1)(1) - 8(2)(1)^2]j +$
$[(1)(2) - 8(2)^2(1)]k$

$= (12 + 2)i + (3 + 1 - 16)j + [2 - 32]k$

$\nabla \cdot A$

$\nabla = \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right)\left(x^2y\,i + (xy + yz)j + xz^2k\right)$

$i \cdot i = j \cdot j = k \cdot k = 1$

$i \cdot j = j \cdot k = k \cdot i = 0$

$\nabla \cdot A = \frac{\partial}{\partial x}(x^2y)i \cdot i + \frac{\partial}{\partial y}(xy + yz)j \cdot j + \frac{\partial}{\partial z}(xz^2)k \cdot k$

$= 2xy + (x + z) + 2xz$

at point $1, 2, 1$

$\nabla \cdot A = 2(1)(2) + [1 + 1] + 2(1)(1) = 4 + 2 + 2$

$= 8$

$\nabla \times B$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial z}(-3xz)\right]i - \left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(yz)\right]j$$

$$+ \left[\frac{\partial}{\partial x}(-3xz) - \partial/\partial y(yz)\right]$$

$$= [2x + 3x]i - [2y - y]j + [-3z - z]k$$

$$\nabla \times B = 5xi - yj - 4zk$$

at point $(1, 2, 1)$

$$\nabla \times B = 5(1)i - (2)j - 4(1)k$$

$$= 5i - 2j - 4k = (5, -2, -4)$$

grad div A

$$A = x^2yi + (xy + yz)j + xz^2 k$$

$$\text{div}A = \frac{\partial(x^2y)}{\partial x} + \frac{\partial(xy + yz)}{\partial y} + \frac{\partial(xz^2)}{\partial z}$$

$$\text{div}A = 2xy + (x + z) + 2xz$$

grad div A = grad Q = $\nabla Q$

$$\nabla Q = \frac{\partial Q}{\partial x} i + \frac{\partial Q}{\partial y} j + \frac{\partial Q}{\partial z} k$$

$$\nabla Q = (2y + 1 + 2z)i + (2x)j + (1 + 2x)k$$

at point $(1, 2, 1)$

$$\nabla Q = (2(1) + 1 + 2(1))i + 2(1)j + (1 + 2(1))k$$

$$\nabla Q = 5i + 2j + 3k$$

grad div A $= 5i + 2j + 3k = (5, 2, 3)$

$$A = x^2yi + (xy + yz)j + xz^2k$$

$$\text{curl}A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \partial/\partial z \\ x^2y & (xy + yz) & xz^2 \end{vmatrix}$$

$i\left[\partial/\partial y(\cancel{xz}^2) - \partial/\partial z(xy+yz)\right] - j\left[\partial/\partial x(xz^2) - \partial/\partial z(x^2y)\right]$

$+ k\left[\partial/\partial x(xy+yz) - \partial/\partial y(x^2y)\right]$

curl $A = [0-y]i - [z^2-0]j + [y-x^2]k - (-y, -z^2, y-x^2)$

at $(1,2,1) \Rightarrow -(2)i - (1^2)j + (2-1)k$

curl $A = -2i - j + k \Rightarrow (-2, -1, 1)$

$(-2, -1, 1)$ //

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