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16/ENG03/037.

CIVIL - ENGR.

COURSE : ENG 282

1.) Mathematical modelling is the process of translating problems from an application area into mathematical formulations whose theoretical and numerical analysis provides insight, answers and guidance useful for the originating application

ii.) a.) Using balance law - law of conservation of mass

b.) Forming a differential equation from an existing algebraic equation of the system

2.) $r = (t^2+3t)i - 2\sin 3t\,j + 3e^{2t}k$

(i) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\,j + 6e^{2t}k$

(ii) $\frac{d^2r}{dr^2} = 2i + 18\sin 3t\,j + 12e^{2t}k$.

(iii) $\frac{d^2r}{dt^2}$ at $t=0$

$= 2i + 18\sin 3(0)j + 12e^{2(0)}k$

$= 2i + 18\sin 0j + 12e^{0}k$

$= 2i + 0j + 12k$

$= \cancel{2i + 2k}$

$\left|\frac{d^2r}{dt^2}\right|$ at $t=0$ $= \sqrt{2^2+0^2+12^2}$

$= \sqrt{148} = 12.17$

5.)

$A = x^2y\,i + (xy + yz)j + xz^2k$

$B = yz\,i - 3xz\,j + 2xy\,k.$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

pt $(\underset{x}{1}, \underset{y}{2}, \underset{z}{1})$.

(i) $\nabla\phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)k$

At $(1,2,1) = (6(1)(2) + (2)(1))i + (3(1)^2 + (1)(1) - 8(2)(1)^2)j + ((1)(2) - 8(2)^2(1))k$

$= (14)i - 12j - 30k$

(ii) $\nabla a = 2xy + (x + z)j + 2xz$

$= 2(1\times2) + (1+1)j + (2\times1\times1)$

$= 4 + 2 + 2$

$= 8$

(iii) $\nabla \times B = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$

$\Rightarrow i\left[d/dy(2xy) - d/dz(-3xz)\right] - j\left[d/dx(2xy) - d/dz(yz)\right] + k\left[d/dx(-3xz) - d/dy(yz)\right]$

$= i(2x + 3x) - j(2y - y) + k(-3z - z)$

$= i(5x) - j(y) + k(-4z)$

$= 5xi - yj - 4zk.$

At $(1,2,1)$

$\Rightarrow 5(1)i - (2)j - 4(1)k$

$= 5i - 2j - 4k$

(iv) Gradient A

$\operatorname{div}A = \nabla\cdot A = \left[di/dx + dj/dy + dk/dz\right]\left[x^2y + (y + yz)j + xz^2k\right]$

$\operatorname{div}A = 2xy + x + y + 2xz$

$\operatorname{grad}(\operatorname{div}A) = \nabla(2xy + x + y + 2xz)$

$\cancel{= \frac{\partial}{dx}(2xy + x + y + 2xz)i + \partial}$

$\operatorname{grad}\operatorname{div}A = (2y + 1 + 2z)i + (2x + 1)j + (2x)k$

At $(1,2,1) = (2(2) + 1 + 2(1))i + (2(1) + 1)j + (2(1))k$

$= 7i + 3j + 2k$

v) Curl Curl A

$$\nabla \times A = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= i\left[\frac{d}{dy}(xz^2) - \frac{d}{dz}(xy+yz)\right] - j\left[\frac{d}{dx}(xz^2) - \frac{d}{dz}(x^2y)\right]$$

$$+ k\left[\frac{d}{dx}(xy+yz) - \frac{d}{dy}(x^2y)\right]$$

$$= i[-y] - j[z^2] + k[y - x^2]$$

$$= -yi - z^2j + (y - x^2)k$$

$$\text{Curl curl } A = \begin{vmatrix} i & j & k \\ d/dx & d/dy & d/dz \\ -y & -z^2 & y-x^2 \end{vmatrix}$$

$$= i\left[\frac{d}{dy}(y-x^2) - \frac{d}{dz}(-z^2)\right] - j\left[\frac{d}{dx}(y-x^2) - \frac{d}{dz}(-y)\right]$$

$$+ k\left[\frac{d}{dx}(-z^2) - \frac{d}{dy}(-y)\right]$$

$$= i(1+2z) - j(-2x) + k(1)$$

$$= i(1+2z) + 2xj + k$$

$$\text{At } (1,2,1) = i(1+2(1)) + 2(1)j + k$$

$$= 3i + 2j + k$$