JASPER VICTORY
16/ENG03/033
CIVIL ENGINEERING
ENG 282

1.) Mathematical modelling is the process of setting up the model, solving it mathematically and interpreting the results in physical or other terms.

(ii) Methods of obtaining models for engineering systems
a.) Mixing problems
b) Exponential growth.

2.) If $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t}K$.
determine (i) $\frac{dr}{dt}$ (ii) $\frac{d^2r}{dt^2}$ and (iii) the value of $\left|\frac{d^2r}{dt^2}\right|$ at $t=0$

Solution

$$r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t}K.$$

i) $\frac{dr}{dt} = (2t + 3)i - (6\cos 3t)j + (6e^{2t})K$

ii) $\frac{d^2r}{dt^2} = 2i + (18\sin 3t)j + (12e^{2t})K$

iii) $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0)j + 12e^{2(0)}K$

$= 2i + 12K.$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{2^2 + 12^2}$

$= \sqrt{4 + 144}$

$= \sqrt{148}$

$= 12.2$ units.

3.) If $A = x^2y\,i + (xy + yz)j + xz^2k$
$B = yz\,i - 3xz\,j + 2xy\,k$, and
$\phi = 3x^2y + xyz - 4y^2z^2 - 3.$
Determine at the point $(1, 2, 1)$
i) $\nabla\phi$, (ii) $\nabla\cdot A$ (iii) $\nabla\times B$ (iv) grad div A and (v) curl curl A

Solution.

$A = x^2y + (xy + yz)j + xz^2k$

$B = yzi - 3xzj + 2xyk$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

i) $\nabla\phi = \left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right]\phi$

$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}$

$\nabla\phi = i[yz + 6xy] + j[3x^2 + xz - 8yz^2] + k[xy - 8y^2z]$

at point $(1, 2, 1)$

$\nabla\phi = i[2\times1 + 12] + j[3\times1^2 + 1\times1 - 8\times2\times1^2] + k[1\times2 - 8\times2^2\times1]$

$\nabla\phi = i(14) + j(-12) + k(-30)$

$\nabla\phi = \cancel{\phi}\,14i - 12j + 30k.$

ii) $\nabla \cdot A$

$\nabla = i\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$

$A = a_x i + a_y j + a_z k.$

$\nabla \cdot A = \left[i\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right] \cdot \left[\cancel{a_x i} + a_y j + a_z k\right]$

$\nabla \cdot A = \frac{\partial}{\partial x} \cdot a_x + \frac{\partial}{\partial y} \cdot a_y + \frac{\partial}{\partial z} \cdot a_z$

$= 2xy + (x+z) + 2xz$

$\nabla \cdot A$ at $(1, 2, 1)$

$\nabla \cdot A = 2\times1\times2 + (1+1) + 2\times1\times1$

$\nabla \cdot A = 4 + 2 + 2$

$\nabla \cdot A = 8$

iii) $\nabla \times B = \begin{bmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -3xz & 2xy \end{bmatrix}$

$$i\begin{vmatrix}\frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ -3xz & 2xy\end{vmatrix} - j\begin{vmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial z}\\ yz & 2xy\end{vmatrix} + k\begin{vmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial y}\\ yz & -3x2\end{vmatrix}$$

$$i\left(\frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial z}(-3xz)\right) - j\left(\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(yz)\right) + k\left(\frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}(yz)\right)$$

$= (2x + 3x)i - j(2y - y) + k(-3z - z)$

$\nabla \times B$ at $(1,2,1)$

$\nabla \times B = (2\times1 + 3\times1)i - j(2\times2 - 2) + k(-3\times1 - 1)$

$\nabla \times B = 5i - 2j - 4k.$

Grad of div A

$\nabla \cdot A = 2xy + (x+z) + 2xz$

$\nabla \cdot A = \left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right]$

$= \left[(2y + 1 + 2z)i + (2x)j + (1 + 2x)k\right]$

$\nabla(\nabla \cdot A)$ at $(2,2,1)$

$\nabla(\nabla \cdot A) = (2\times2 + 1 + 2\times1)i + (2\times1)j + (1+2\times1)k$

$= (7)i + (2)j + 3k.$

$= 7i + 2j + 3k.$

Curl Curl A

$$\text{Curl } A = \nabla \times A = \begin{vmatrix} i & j & k\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ x^2y & (xy+yz) & (xz^2)\end{vmatrix}$$

$$i\begin{vmatrix}\frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ (xy+yz) & (xz^2)\end{vmatrix} - j\begin{vmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial z}\\ x^2y & (xz^2)\end{vmatrix} + k\begin{vmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial y}\\ x^2y & (xy+yz)\end{vmatrix}$$

$$i\left(\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy+yz)\right) - j\left(\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}x^2y\right) + k\left(\frac{\partial}{\partial x}(xy+y^2) - \frac{\partial}{\partial y}(x^2y)\right)$$

$i(0 - y) - j(z^2 - 0) + k(y - x^2)$

$$\nabla \times A = -y\,i - z^2\,j + (y - x^2)\,k.$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$i\left|\frac{\partial}{\partial y}(y-x^2) - \frac{\partial}{\partial z}(-z^2)\right| - j\left|\frac{\partial}{\partial x}(y-x^2) - \frac{\partial}{\partial z}(-y)\right| + k\left|\frac{\partial}{\partial x}(-z^2) - \frac{\partial}{\partial y}(-y)\right|$$

$$i(1 + 2z) - j(-2x + 0) + k(0 + 1)$$

$\nabla \times (\nabla \times A)$ at $(1, 2, 1)$

$$\nabla \times (\nabla \times A) = i(1 + 2\times 1) - j(-2\times 1) + k(1)$$

$$\nabla \times (\nabla \times A) = 3i + 2j + k.$$