iv) grad div A

grad (2xy + (x+z) + 2xz)

Let div A = C = ∇A

$\nabla(\nabla A) = \nabla C = i\frac{\partial C}{\partial x} + j\frac{\partial C}{\partial y} + k\frac{\partial C}{\partial z}$

$= i(2y + 1 + 2z) + j(2x) + k(1+2x)$

At (1, 2, 1)

$\nabla C = i(2(2) + 1 + 2(1)) + j(2(1)) + k(1 + (2)(1))$

$= i(4 + 1 + 2) + j(2) + k(1+2)$

$= 7i + 2j + 3k$

v) Curl Curl A

Curl A . $\nabla \times A$

$$= \begin{bmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{bmatrix}$$

$= i(0-y) - j(z^2-0) + k(y-x^2)$

$= yi - z^2j + k(y-x^2)$

At (1, 2, 1)

Curl A = $-2i - j + k$

Curl Curl A = $\nabla + (\nabla \times A)$

$$\nabla \times (\nabla \times A) \begin{bmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2) \end{bmatrix}$$

$= i(1+2z) - j(-2x-0) + k(0+1)$

$= i(1+2z) + 2x^2j + k$

At point (1, 2, 1)

$\nabla \times (\nabla \times A) = i(1+2(1)) + 2(1)^2j + k$

$= 3i + 2j + k$

$\frac{\partial \phi}{\partial z} = xy - 8y^2z$

$\frac{\partial \phi}{\partial y} = 3x^2 + xz - 8yz^2$

At (1, 2, 1)

$\frac{\partial \phi}{\partial x} = 6(1)(2) + (2)(1) = 12 + 2 = 14$

$\frac{\partial \phi}{\partial y} = 3(1)^2 + (1)(1) - 8(2)(1)^2 = 3 + 1 - 16 = -12$

$\frac{\partial \phi}{\partial z} = (1)(2) - 8(2)^2(1) = 2 - 32 = -30$

$\nabla \phi = 14i - 12j - 30k$

ii) $\bar{\nabla} \cdot A = \frac{\partial a}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$A = a_x i + a_y j + a_z k$

$\bar{\nabla} \cdot A = 2xy + (x+z) + 2xz$

At (1, 1, 1)

$\bar{\nabla} \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$= 4 + 2 + 2 = 8$

iii) $\bar{\nabla} \times B$

$$\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$= i(2x + 3x) - y(2y - y) + k(-3z - z)$

$= 5xi - yj - 4zk$

At (1, 2, 1)

$\bar{\nabla} \times B = 5i - 2j - 4k$.

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16/ENG05/1012
Mechatronics

Assignment

1a) A mathematical model is a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical and other terms

b i Exponential growth / decay (Use of ODE)
ii mixing problems

2) $r = (t^2+3t)i - 2\sin 3t\,j + 3e^{2t}k$

i) $\frac{dr}{dt} = (2t+3i) - 6\cos 3t\,j + 6e^{2t}k$

ii) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t\,j + 12e^{2t}k$

iii) $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 12k$

iv) $\left|\frac{d^2r}{dx}\right| = \sqrt{2^2+12^2} = \sqrt{148} = 2\sqrt{37} = 12.17$

$S = A = x^2y\,i + (xy+y^2)j + xz^2k$

$B = yz\,i + 3x\,zj + 2xyk$

$\phi = 3x^2y + xyz^2 - 4y^2z^2 - 3$

$\Rightarrow \nabla\phi = \frac{\partial di}{\partial x} + \frac{\partial d}{\partial y}j + \frac{\partial B}{\partial z}$

$\frac{\partial\phi}{\partial x} = 6xy + y^2$