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COURSE: ENG 282
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1.> i) Mathematical modelling is the process of setting up a model, solving it mathematically, and interpreting the result in physical or in order terms.

ii) a. Using Balance law - Law of conservation of mass

b. forming a differential equation from an existing algebraic equation of the system.

2.> $r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t} k.$

i) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t} k.$

ii) $\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right) =$

$\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t} k.$

$\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0)\, j + 12e^{2(0)} k.$

$= 2i + 18\sin 0\, j + 12e^0 k$

$= 2i + 18\times 0\, j + 12\times 1\, k$

$= \underline{2i + 12k}$

iv) $\left|\frac{d^2r}{dt^2}\right| = |2i + 12k|$

$= \sqrt{(2i)^2 + (12k)^2}$

$= \sqrt{4\times 1\cdot i + 144\times k\cdot k} = \sqrt{4\times 1 + 144\times 1}$

$= \sqrt{4+144} = \sqrt{148}.$

v) $\left|\frac{d^2r}{dt^2}\right|_{t=0} = 2\sqrt{37}$

$\left|\frac{d^2r}{dt^2}\right| = 2\sqrt{37} = 12.16552506$

$\simeq \underline{12.17}$

3.) at point $(1, 2, 1)$

$A = x^2y\,i + (xy + yz)\,j + xz^2 k.$

$B = yz\,i - 3xz\,j + 2xy\,k.$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

i.) $\underline{\nabla\phi} = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{\partial y} j + \frac{\partial}{\partial z}\phi k$

$\nabla\phi = \frac{\partial(3x^2y + xyz - 4y^2z^2 - 3)}{\partial x} i + \frac{\partial(3x^2y + xyz - 4y^2z^2 - 3)}{\partial y} j + \partial(3x^2y$

$+ \frac{\partial(3x^2y + xyz - 4y^2z^2 - 3)}{\partial z} k$

$\nabla\phi = (6xy + yz)\,i + (3x^2 + xz - 8yz^2)\,j + (xy - 8y^2z)\,k.$

at point $(1, 2, 1)$

$\nabla\phi = (6(1)(2) + (2)(1))\,i + (3(1^2) + (1)(1) - 8(2)(1^2))\,j + ((1)(2) - 8(2^2)(1))k$

$\nabla\phi = (12 + 2)\,i + (3 + 1 - 16)\,j + (2 - 32)\,k$

$\nabla\phi_{(1,2,1)} = 14i - 12j - 30k$

ii.) $\underline{\nabla \cdot A} = \frac{\partial}{\partial x} A i + \frac{\partial}{\partial y} \cdot A j + \frac{\partial}{\partial z} \cdot A k.$

$\nabla A = \frac{\partial}{\partial x}(x^2y)\,i \cdot i +)0$

$\nabla A = 2xy\; i \cdot i + (x + z)\, j \cdot j + 2xz\; k \cdot k$

$= 2xy + (x + z) + 2xz$

$\nabla A(1, 2, 1) = 2(1)(2) + (1)+(1) + 2(1)(1)$

$= 4 + 2 + 2 = 8$

$\nabla A_{1,2,1} = \underline{8}$

iii.) $\underline{\nabla \times B} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$

$= \left(\frac{\partial}{\partial y} \cdot 2xy - \frac{\partial}{\partial z} \cdot -3xz\right) i - j\left(\frac{\partial}{\partial x} \cdot 2xy - \frac{\partial}{\partial z} \cdot yz\right) + k\left(\frac{d}{\partial x} \cdot -3xz - \frac{\partial}{\partial y} \cdot yz\right)$

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$$\nabla\times B = (2x-(-3x))i \quad -j(2y-y) \quad +k\,(\overset{-3z}{\cancel{-6x}} - z)$$

$$\nabla\times B = 5x\,i \quad -yj \quad -k\,(\overset{4z}{\cancel{-6x+(4)}})$$

$$\nabla\times B = 5xi - yj - 4zk.$$

1,2,1 $= \cancel{5(1)i - y}$

$5(1)i - (2)j - 4(1)k$

$= \underline{5i - 2j - 4k}$

iv). $\underline{\text{grad.div } A} = \nabla \cdot \nabla A$

$$= \frac{\partial}{\partial x}\cdot\nabla A\, i + \frac{\partial}{\partial y}\cdot\nabla A\, j + \frac{\partial}{\partial z}\cdot\nabla A\, k.$$

recall $\nabla A = 2xy + x + z + 2xz$

~~R.Rea~~ $\nabla\cdot\nabla A = \frac{\partial}{\partial x}(2xy+x+z+2xz)\cdot i + \frac{\partial}{\partial y}(2xy+x+z+2xz)j + \frac{\partial}{\partial z}(2xy+x+z+2xz)k$

$$\nabla\cdot\nabla A = (2y+1+2z)\cdot i + (2x)j + (1+2x)k$$

at point (1,2,1)

$= 2(2)+1+2(1)i + 2(1)j + 1+2(1)k$

$(4+1+2)i + 2j + (1+2).k$

$\nabla\cdot\nabla A = 7i + 2j + 3k.$

grad. div $A = \underline{7i + 2j + 3k.}$

v) ~~curl A.~~ $\underline{\text{curl curl } A}$.

$$\text{curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$i\left(\frac{\partial}{\partial y}\cdot xz^2 - \frac{\partial}{\partial z}\cdot(xy+yz)\right) - j\left(\frac{\partial}{\partial x}\cdot xz^2 - \frac{\partial}{\partial z}\cdot x^2y\right) + \cancel{k\left(\ldots\right)}$

$+k\left(\frac{\partial (xy+yz)}{\partial x} - \frac{\partial}{\partial y}x^2y\right)$

$i\cdot(0-y) - j(z^2 - 0) \cancel{+k(z^2-(x+z))} + k(y - x^2)$

$\text{curl } A = yi - z^2j + (y-x^2)k,$

~~curl curl A = ... i j ∂/ ∂/∂x~~

curl curl A = curl (curl A)

curl A = $\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$

curl A $= i \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy+yz) & xz^2 \end{vmatrix} - j \begin{vmatrix} \partial/\partial z & \partial/\partial z \\ x^2y & xz^2 \end{vmatrix} + k \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2y & (xy+yz) \end{vmatrix}$

curl A $= i\left(\frac{\partial}{\partial y} \cdot xz^2 - \frac{\partial}{\partial z}(xy+yz)\right) - j\left(\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right)$

$+ k\left(\frac{\partial}{\partial x}(xy+yz) - \frac{\partial}{\partial y}(x^2y)\right)$

$= i(0-y) + -j(z^2-0) + k(y-x^2)$

curl A $= -yi - z^2j + (y-x^2)k.$

curl A $= \underline{yi - z^2j + (y-x^2)k}$

curl curl A = $\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -yi & -z^2 & y-x^2 \end{vmatrix}$

$i\left(\frac{\partial}{\partial y} \cdot (y-x^2) - \frac{\partial}{\partial z} \cdot -z^2\right) - j\left(\frac{\partial}{\partial x} \cdot y-x^2 - \frac{\partial}{\partial z} \cdot y\right)$

$+ k\left(\frac{\partial}{\partial x} \cdot z^2 - \frac{\partial}{\partial y} \cdot (-y)\right)$

curl curl A $= \cancel{-2x^2 - 2z}$

$= (1+2z)i - j(-2x - 0) + k(0-(-1))$

curl curl A $= (1+2z)i + 2xj + k.$

curl curl A $= (1+2z)i + 2xj + k$

$= (1+2(1))i + 2(1)j + k$

$= 3j + 2j + k = 3i + 2j + k$

curl curl A $= \underline{3i + 2j + k}$