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16/ENG04/1031
Elect/Elect Eng..
ENG 282 Engr. Maths II

Assignment:

1:) Mathematical modelling is the process of translating problems from an application area into mathematical formulations whose theoretical and numerical analysis provides insights, answers & guidance useful to the originating application.

ii:) a.) Using balance law → Law of conservation of mass.
b.) Forming a differential equation from an existing algebraic equation of the system.

2.) $r = (t^2+3t)i - 2\sin 3t\,j + 3e^{2t}k$.

i.) $\frac{\partial r}{\partial t} = (2t+3)i - 6\cos 3t\,j + 6e^{2t}k$.

ii.) $\frac{\partial^2 r}{\partial t^2} = 2i + 18\sin 3t\,j + 12e^{2t}k$.

iii.) $\frac{d^2r}{dt^2}$ at $t=0$

$= 2i + 18\sin 3(0)j + 12e^{2(0)}k$.

$= 2i + 18\sin 0\,j + 12e^{0}k$.

$= 2i + 0j + 12k$.

$\left|\frac{\partial^2 r}{\partial t^2}\right|_{t\to 0} = \sqrt{2^2+0^2+12^2} = \sqrt{148} = 12.17$.

3.) $A = x^2yi + (xy+yz)j + xz^2k$.

$B = yzi - 3xzj + 2xyk$.

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$.

Pt (1, 2, 1)

x y z

i.) $\Delta\phi = (6xy+yz)i + (3x^2 + x_2 - 8yz^2)j + (xy - 8y^2z)k$.

At (1, 2, 1) $= (6\times1\times2 + 2\times1)i + (3(1)^2 + 1\times1 - 8(2)(1)^2)j + (1\times2 - 8(2)^2(1))k$.

$= 14i - 12j - 30k$

ii.) $\Delta \cdot A = 2xy + (x+z) + 2xz$

At (1, 2, 1) $= (2\times1\times2) + (1+1) + (2\times1\times1)$.

$= 4 + 2 + 2$

$= 8$.

iii.) $\Delta \cdot B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial k \\ yz & -3xz & 2xy \end{vmatrix}$

$= i\left[\frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial z}(-3xz)\right] - j\left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(yz)\right]$

$+ k\left[\frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}(yz)\right]$

$= i[2x + 3x) - j(2y - y] + k[-3z - z]$.

$= 5xi - yj - 4zk$.

At (1, 2, 1) $= 5(1)i - 2j - 4(1)k$

$= 5i - 2j - 4k$.

iv.) grad div A

div $A = 2xy + (x+y) + 2xz$

grad div $A = (2y+y+2z)i + (2x+x)j + (2x)k$.

$= (3y+2z)i + (3x)j + 2xk$.

$$\text{At } (1,2,1) = (3\times 2 + 2\times 1)i + 3(1)j + 2(1)k$$
$$= 8i + 3j + 2k.$$

v.) Curl Curl A

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy+yz)\right] - j\left[\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right] +$$

$$k\left[\frac{\partial}{\partial x}(xy+yz) - \frac{\partial}{\partial y}(x^2y)\right].$$

$$= i[-(xy+y)] - j[z^2] + k[(y+yz) - x^2].$$

$$= -(xy+y)i - z^2 j + [(y+yz) - x^2]k.$$

$$\text{Curl Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -(xy+y) & -z^2 & (y+yz)-x^2 \end{vmatrix}$$

$$= i\left[\frac{\partial[(y+yz)-x^2]}{\partial y} - \frac{\partial}{\partial z}(-z^2)\right] - j\left[\frac{\partial[(y+yz)-x^2]}{\partial x} - \frac{\partial}{\partial z}(-(xy+y))\right]$$

$$+ k\left[\frac{\partial}{\partial x}(-z^2) - \frac{\partial}{\partial y}[-(xy+y)]\right].$$

$$= i[z - x^2 + 2z] - j[-2x] + k[x].$$

$$= i[3z - x^2] + 2xj + xk$$

$$= [3z - x^2]i + 2xj + xk$$

$$\text{At } (1,2,1) = (3\times 1 - (1)^2)i + 2(1)j + (1)k$$
$$= 2i + 2j + k.$$