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ELECTRICAL/ELECTRONICS
ENG 282 - ASSIGNMENT 7

Question 1

i) Define mathematical modelling

Mathematical modeling can be defined as the process of setting up a model, solving it mathematically and interpreting the results in physical or other terms.

ii) Outline two methods for obtaining models for engineering systems.

Question 2

If $r = (t^2 + 3t)i - 2\sin 3t j + 3e^{2t}k$,

determine

(i) $\frac{dr}{dt} = \frac{d}{dt}\left[(t^2+3t)i - 2\sin 3t j + 3e^{2t}k\right]$

$$\frac{dr}{dt} = (2t+3)i - 6\cos 3t j + 6e^{2t}k$$

(ii) $\frac{d^2r}{dt^2} = \frac{d}{dt}\left[(2t+3)i - 6\cos 3t j + 6e^{2t}k\right]$

$$\frac{d^2r}{dt^2} = 2i + 18\sin 3t j + 12e^{2t}k$$

(iii) the value of $\left|\frac{d^2r}{dt^2}\right|$ at $t=0$ $= \sqrt{(2)^2 + (18\sin 3(0))^2 + (12e^{2\times 0})^2}$

$$= \sqrt{4 + (18\times 0)^2 + (12\times 1)^2}$$

$$= \sqrt{4+144}$$

$$= \sqrt{148}$$

$$\therefore \left|\frac{d^2r}{dt^2}\right| = 12.17$$

Question 3

If $A = x^2y\,i + (xy + yz)\,j + xz^2k$

$B = yz\,i - 3xz\,j + 2xy\,k$, and

$\phi = 3x^2y + xyz\,j - 4y^2z^2 - 3$

determine, at the point $(1,2,1)$

$$\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$$

$$= \frac{\partial(3x^2y + xyz - 4y^2z^2 - 3)}{\partial x}i + \frac{\partial(3x^2y + xyz - 4y^2z^2 - 3)}{\partial y}j + \frac{\partial(3x^2y + xyz - 4y^2z^2 \ldots)}{\partial z}$$

$$= (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)k$$

$$\nabla\phi/_{(1,2,1)} = (6(1)(2) + (2)(1))i + (3(1)^2 + (1)(1) - 8(2)(1)^2)j + ((1)(2) - 8(2)^2(1))k$$

$$\nabla\phi/_{(1,2,1)} = (12+2)i + (3+1-16)j + (2-32)k$$

$$\nabla\phi/_{(1,2,1)} = 14i - 12j - 30k$$

(ii) $$\nabla\cdot A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right)\cdot(x^2y\,i + (xy + yz)j + xz^2k)$$

$$= \frac{\partial x^2y}{\partial x} + \frac{\partial(xy + yz)}{\partial y} + \frac{\partial xz^2}{\partial z}$$

$$\nabla\cdot A = 2xy + x + z + 2xz$$

$$\nabla\cdot A/_{(1,2,1)} = 2(1)(2) + 1 + 1 + 2(1)(1)$$

$$= 4 + 2 + 2$$

$$\nabla\cdot A/_{(1,2,1)} = 8$$

(iii) $$\nabla\times B = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right)\times(yz\,i - 3xz\,j + 2xy\,k)$$

$$\nabla\times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$$= i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ -3xz & 2xy \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ yz & 2xy \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ yz & -3xz \end{vmatrix}$$

$$= i\left(\frac{\partial 2xy}{\partial y} + \frac{\partial 3xz}{\partial z}\right) - j\left(\frac{\partial 2xy}{\partial x} - \frac{\partial yz}{\partial z}\right) + k\left(\frac{-\partial 3xz}{\partial x} - \frac{\partial yz}{\partial y}\right)$$

$$= (2x + 3x)i - (2y - y)j + (-3z - z)k$$

$$\nabla\times B = 5x\,i - y\,j - 4z\,k$$

$$\nabla\times B/_{(1,2,1)} = 5(1)i - 2(j) - 4(1)k$$

$\nabla\times B\Big|_{(1,2,1)} = 5i - 2j - 4k$

iv) Grad div A

$\text{div}A = \nabla\cdot A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right)\cdot\left(x^2 y\, i + (xy + yz)j + xz^2 k\right)$

$\text{div}A = 2xy + x + y + 2xz$

$\text{grad}(\text{div}A) = \nabla(2xy + x + y + 2xz) = \frac{\partial(2xy + x + y + 2xz)}{\partial x}i + \frac{\partial(2xy + x + y + 2xz)}{\partial y}j$
$+ \frac{\partial(2xy + x + y + 2xz)}{\partial z}k$

$\text{grad div}A = (2y + 1 + 2z)i + (2x + 1)j + (2x)k$

$\text{grad div}A\Big|_{(1,2,1)} = (2(2) + 1 + 2(1))i + (2(1) + 1)j + (2(1))k$

$= (4 + 1 + 2)i + (2 + 1)j + 2k$

$\text{grad div}A\Big|_{(1,2,1)} = 7i + 3j + 2k$

(v) Curl Curl A = Curl(Curl A)

$\text{Curl }A = \nabla\times A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial z}{\partial k}\right)\times\left(x^2 y\, i + (xy + yz)j + xz^2 k\right)$

$\text{Curl }A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & xy+yz & xz^2 \end{vmatrix}$

$= i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy+yz) & xz^2 \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2y & xz^2 \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2y & (xy+yz) \end{vmatrix}$

$= \left(\frac{\partial xz^2}{\partial y} - \frac{\partial(xy+yz)}{\partial z}\right)i - \left(\frac{\partial xz^2}{\partial x} - \frac{\partial x^2y}{\partial z}\right)j + \left(\frac{\partial(xy+yz)}{\partial x} - \frac{\partial x^2y}{\partial y}\right)k$

$\text{Curl }A = yi - z^2 j + (y - x^2)k$

$\text{Curl Curl }A = \text{Curl}\left(yi - z^2 j + (y - x^2)k\right)$

$= \nabla\times\left(yi - z^2 j + (y - x^2)k\right)$

$= \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right)\times\left(yi - z^2 j + (y - x^2)k\right)$

$\text{Curl Curl }A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & y - x^2 \end{vmatrix}$

$= i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ -z^2 & (y - x^2) \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ -y & y - x^2 \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ -y & -z^2 \end{vmatrix}$

$= \left(\frac{\partial(y - x^2)}{\partial y} + \frac{\partial z^2}{\partial z}\right)i - \left(\frac{\partial(y - x^2)}{\partial x} + \frac{\partial y}{\partial z}\right)j + \left(\frac{-\partial z^2}{\partial x} + \frac{\partial y}{\partial y}\right)k$

$= (1 + 2z)i - (-2x)j + (1)k$

$= (1 + 2z)i + 2xj + k$

$= (1 + 2(1))i + 2(1)j + k$

$\text{Curl Curl }A = 3i + 2j + k$