Thank God Clinton
16/ENG06/068
Mechanical Engineering
ENG 282

1 mathematical Modelling
This is the Process of Setting up a model, Solving it mathematically, and Interpreting the result in Physical or in order terms

ii using balance law - law of conservation of mass
b Forming a differential equation from an existing algebraic equation of the System.

2 $r = (t^2 + 3t)i - 2\sin 3t\, j + 3e^{2t} k$

$$\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t} k.$$

$$\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right)$$

$$\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t} k.$$

$$\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0)\, j + 12e^{2(0)} k.$$

$$= 2i + 18\sin 0\, j + 12e^0 k$$

$$2i + 18 \times 0\, j + 12 \times 1\, k$$

$$= 2i + 12k$$

iii $$\left|\frac{d^2r}{dt^2}\right| = |2i + 12k|$$

$$= \sqrt{(2i)^2 + (12k)^2}$$

$$= \sqrt{4 \times i \cdot i + 144 \times k \cdot k}$$

$$= \sqrt{148}$$

$$= 2\sqrt{37}$$

3) at point (1, 2, 1)

$A = x^2 y i + (xy + yz) j + xz^2 k.$

$B = yz i - 3xz j + 2xy k.$

$\phi = 3x^2 y + xyz - 4y^2 z^2$

$$\bar{\nabla}\phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k$$

$$\bar{\nabla}\phi = \frac{\partial (3x^2 y + xyz - 4y^2 z^2 - 3)}{\partial x} i + \frac{\partial (3x^2 y + xyz - 4y^2 z^2 - 3)}{\partial y} j + \frac{\partial (3x^2 y + xyz - 4y^2 z^2 - 3)}{\partial z} k$$

$\bar{\nabla}\phi = (6xy + yz) i + (3x^2 + xz - 8yz^2) j + (xy - 8y^2 z) k.$

at point 1, 2, 1

$\bar{\nabla}\phi = (6(1)(2) + (1)(1)) i + (3(1^2) + (1)(1) - 8(2)(1^2)) j + ((1)(2) - 8(2^2)(1))$ k

$\bar{\nabla}\phi_{(1,2,1)} = 14 i - 12 j - 30 k$

ii) $\bar{\nabla} \cdot A = \frac{\partial}{\partial x} A i + \frac{\partial}{\partial y} \cdot A j + \frac{\partial}{\partial z} \cdot A k$

$\bar{\nabla} A = 2xy\, i \cdot i + (x + z) j \cdot j + 2xz\, k \cdot k$

$= 2xy + (x + z) + 2xz$

$\bar{\nabla} A (1, 2, 1) = 2(1)(2) + (1) + (1) + 2(1)(1)$

$4 + 2 + 2 = 8$

$\bar{\nabla} A = 8$

iii) $\bar{\nabla} \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$

$= \left(\frac{\partial}{\partial y} \cdot 2xy - \frac{\partial}{\partial z} - 3xz\right) i - j \left(\frac{\partial}{\partial x} \cdot 2xy - \frac{\partial}{\partial z} \cdot yz\right) + k$

$\left(\partial/\partial x \cdot -3z - \partial/\partial y \cdot yz\right)$

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$\nabla \times B = (2x - (-3x))i - j(2y - y) + k(-3z - z)$

$\nabla \times B = 5xi - yj - k(4z)$

$\nabla \times B = 5xi - yj - 4zk.$

$1, 2, 1 = 5(1)i - (2)j - 4(1)k$

$= 5i - 2j - 4k$

$\text{grad} \cdot \text{div}\, A = \bar{\nabla} \cdot \bar{\nabla} A$

$= \frac{\partial}{\partial x} \cdot \bar{\nabla} A\, i + \frac{\partial}{\partial y} \cdot \bar{\nabla} A\, j + \frac{\partial}{\partial z} \cdot \bar{\nabla} A\, k$

recall $\bar{\nabla} A = 2xy + x + z + 2xz$

$\bar{\nabla} \cdot \bar{\nabla} A = \frac{\partial}{\partial x}(2xy + x + z + 2xz) \cdot i + \frac{\partial}{\partial y}(2xy + x + z + 2xz)j + \partial/\partial z\, (2xy + x + z + 2xz)k$

$\bar{\nabla} \cdot \nabla A = (2y + 1 + 2z) \cdot i + (2x)j + (1 + 2x)k$

at point $(1, 2, 1)$

$= 2(2) + 1 + 2(1)i + 2(1)j + 1 + 2(1)k$

$(4 + 1 + 2)i + 2j + (1 + 2)k$

$\bar{\nabla} \cdot \bar{\nabla} A = 7i + 2j + 3k$

$\text{grad div } A = 7i + 2j + 3k$

$\text{Curl curl } A = \text{curl}(\text{curl } A)$

$$\text{Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & (xy + yz) & xz^2 \end{vmatrix}$$

$$\text{Curl } A = i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy + yz) & xz^2 \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2 y & xz^2 \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2 y & (xy + yz) \end{vmatrix}$$

Curl A

$$= i\left(\frac{\partial}{\partial y} \cdot xz^2 - \frac{\partial}{\partial z}(xy + yz)\right) - j\left(\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2 y)\right)$$

$+ k\left(\frac{\partial}{\partial x}(xy+yz) - \frac{\partial}{\partial y}(x^2y)\right)$

$= i(0-y) + -j(z^2-0) + k(y-x^2)$

$\text{Curl } A = -yi - z^2 j + (y-x^2)k$

$\text{Curl } A = -yi - z^2 j + (y-x^2)k$

$$\text{Curl Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -yi & -z^2 & y-x^2 \end{vmatrix}$$

$$i\left(\frac{\partial}{\partial y}\cdot(y-x^2) - \frac{\partial}{\partial z} - z^2\right) - j\left(\frac{\partial}{\partial x}\cdot y-x^2 - \frac{\partial}{\partial z}\cdot y\right)$$

$$+ k\left(\frac{\partial}{\partial x}\cdot z^2 - \frac{\partial}{\partial y}\cdot(-y)\right)$$

$\text{Curl Curl } A$

$= (1+2z)i - j(-2x-0) + k(0-(-1))$

$\text{Curl Curl } A = (1+2z)i + 2xj + k$

$\text{Curl Curl } A = (1+2z)i + 2xj + k$

$= (1+2(1))i + 2(1)j + k$

$= 3i + 2j + k = 3i + 2j + k$

$\text{Curl Curl } A = 3i + 2j + k$

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