$\nabla \times \vec{A} = -yi - j(z^2) + k(x^2 - y)$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & x^2 - y \end{vmatrix}$$

$= i(1 + 2z) - j(0 - 2x) + k(1 - 0)$

$\nabla \times (\nabla \times \vec{A}) = (1 + 2z)i + 2xj + k$

$= 1 + 2(1)\,i + 2(1)j + k$

$= \underline{\underline{3i + 2j + k}}$

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1. Question 1

(i) Mathematical modelling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers and guidance useful for the originating application.

(ii) ⓐ In Electrical Engineering → (i) Power Supply
(ii) Network optimization

ⓑ In Chemical Engineering → (i) Chemical equilibrium.

Question 2

$r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t}k$

(i) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t}k$

(ii) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t\, j + 12e^{2t}k.$

(iii) $\left|\frac{d^2r}{dt^2}\right|_{at\ t=0} = 2i + 18\sin 3(0)\, j + 12e^{2(0)}k$

$= 2i + 12k$

$\Rightarrow \sqrt{4+144} = \sqrt{148} = 2\sqrt{37} = \underline{12.165}$

Question 3

$A = x^2y\,i + (xy + yz)\,j + xz^2k$

$B = yz\,i - 3xz\,j + 2xy\,k$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

(i) $\nabla\phi$ at point $(1,2,1)$

$$\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$$

$$\frac{\partial\phi}{\partial x} = 6xy + yz$$

$$\frac{\partial\phi}{\partial y} = 3x^2 + xz - 8yz^2$$

$$\frac{\partial\phi}{\partial z} = xy - 8y^2z$$

$$\therefore \nabla\phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)k$$

at point $(1,2,1)$

$$\nabla\phi = (12+2)i + (3+1-8(2)(1))j + (2-32)k$$
$$= 14i + (4-16)j - 30k$$
$$= 14i - 12j - 30k$$

$\nabla\phi = \underline{14i - 12j - 30k}$.

$\nabla \cdot A$

$$\nabla\cdot A = \frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k \cdot \left(x^2y\,i + (xy + yz)j + xz^2k\right)$$

$$\nabla \cdot A = \frac{\partial (x^2y)}{\partial x} + \frac{\partial}{\partial y}(xy+yz) + \frac{\partial}{\partial z}(xz^2)$$

$$= 2xy + (x+z) + 0 + (2xz)$$

$$= 2(1)(2) + (1+1) + 2\times1\times1 = 6+2$$

$$= 4+2+2$$

$$= \underline{8}$$

$$\therefore \nabla \cdot A = \underline{\underline{8}}$$

(iii) $\nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$

$$i\left(\frac{\partial}{\partial y}(2xy) + \frac{\partial}{\partial z}(3xz)\right) - j\left(\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(yz)\right) + k\left(\frac{\partial}{\partial z}(-3xz) - \frac{\partial (yz)}{\partial y}\right)$$

$$= i(2x+3x) - j(2y-y) + k(-3z-z)$$

$$= i(5x) - j(y) + k(-4z)$$

$$= 5xi - yj - 4zk$$

at point $(1,2,1)$

$$\nabla \times B = \underline{5i - 2j - 4k}$$

iv) grad. div. A = $\nabla \cdot \vec{A}$

Div A = $2xy + (x+z) + 2zx$

$$\nabla(\nabla \cdot A) = \frac{\partial (\nabla A)}{\partial x} i + \frac{\partial (\nabla \cdot A)}{\partial y} j + \frac{\partial (\nabla \cdot A)}{\partial z} k$$