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 DEPT: MECHANICAL ENGINEERING

Answers

1. Mathematical modelling is the art of translating problems from an application area into tractable mathematical formulations whose theoretical and numerical analysis provides insight, answers and guidance useful for the originating application.

1b. In chemical engineering \Rightarrow chemical equilibrium
 In Electrical engineering \Rightarrow power supply network optimization

$$\begin{aligned}
 2. \quad r &= (t^2 + 3t)j - 2\sin 3tj + 3e^{2t}k \\
 \frac{dr}{dt} &= (2t + 3)j - 6\cos 3tj + 6e^{2t}k \\
 \frac{d^2r}{dt^2} &= 2j + 18\sin 3tj + 12e^{2t}k \\
 \left| \frac{d^2r}{dt^2} \right|_{at \ t=0} &= 2j + 18\sin 3tj + 12e^{2t}k \\
 \left| \frac{d^2r}{dt^2} \right|_{at \ t=0} &= 2j + 18\sin(3 \times 0)j + 12e^{2(0)}k \\
 \left| \frac{d^2r}{dt^2} \right| &= \sqrt{4 + 144} = \sqrt{148} \approx 12.1655
 \end{aligned}$$

$$\begin{aligned}
 3. \quad A &= x^2y_1 + (y_1 + y_2)j + xz^2k \\
 B &= y_2i - 3xzj + 2yzk \\
 \phi &= 3yz + 2(y - 4y^2z^2 - 3)
 \end{aligned}$$

1) $\nabla \phi$ at point (1, 3, 1)

$$\nabla \phi = \frac{\partial \phi}{\partial x}i + \frac{\partial \phi}{\partial y}j + \frac{\partial \phi}{\partial z}k$$

$$\frac{\partial \phi}{\partial x} = 6xy + yz$$

$$\frac{\partial \phi}{\partial y} = 3xz + xz - 8y^2z^2$$

$$\frac{\partial \phi}{\partial z} = xy - 8y^2 z$$

$$\nabla \phi = (6xy + yz^2)i + (3x^2 + 12 - 8yz^2)j + (6y - 8y^2 z)k$$

at point (1, 2, 1)

$$\nabla \phi = (12+2)i + (3+1-8(2)(2))j + (2-8(3))k$$

$$= 14i + (4-16)j + (-30)k$$

$$= 14i - 12j - 30k$$

$$\nabla \phi = 14i - 12j - 30k$$

$$\nabla A = \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \cdot (4xy + 6yz + yz^2) + xz^2 k$$

$$\nabla A = \frac{\partial}{\partial x} (4xy) + \frac{\partial}{\partial y} (6yz + yz^2) + \frac{\partial}{\partial z} (xz^2)$$

$$\nabla A = 2xy + (x+z) + 0 + (2xz)$$

$$= 2(1)(2) + (1+1) + (2 \cdot 1 \cdot 2)$$

$$= 4 + 2 + 4$$

$$\nabla A = 8$$

$$\nabla \times B = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -3xz & 2xy \end{vmatrix}$$

$$\left(\frac{\partial}{\partial y} (2xy) + \frac{\partial}{\partial z} (3xz) \right) i - \left(\frac{\partial}{\partial x} (2xy) - \frac{\partial}{\partial z} (yz) \right) j$$

$$+ \left(\frac{\partial}{\partial x} (-3xz) - \frac{\partial}{\partial y} (yz) \right) k$$

$$i(2+3z) - j(2y-y) + k(-3z-2)$$

$$i(5z) - j(y) + k(-3z-2)$$

$$5zi - yj - 4zk$$

at point (1, 2, 1)

$$\nabla \times B = 5i - 2j - 4k$$

iv grad of A $\nabla \cdot A$

$$D_1 A = 2xy + (x+z) + 2xz$$

$$\nabla(\nabla \cdot A) = \frac{\partial}{\partial x} (2xy) i + \frac{\partial}{\partial y} (x+z) j + \frac{\partial}{\partial z} (2xz) k$$

$$\begin{aligned}\nabla(\nabla A) &= i(2y+1+2z) + 2xj + (1+2x)k \\ &= i(4+1+2) + 2j + 3k \\ &= 7i + 2j + 3k\end{aligned}$$

$$\nabla \times (\nabla \times \vec{A}) \quad \vec{A} = x^2y i + (xy + yz) j + xz^2 k$$

$$\nabla \times \vec{A} = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$i(0-y) - j(z^2-0) + k(x^2+y) - yi - j(z^2) + k(x^2-y)$$

$$\nabla \times \vec{A} = -yi - j(z^2) + k(x^2-y)$$

$$\nabla \times (\nabla \times \vec{A}) = \begin{vmatrix} i & j & k \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ -y & -z^2 & x^2-y \end{vmatrix}$$

$$= i(1+2z) - j(0-2x) + k(1-0)$$

$$= (1+2z)i + 2xj + k$$

$$= (1+2(2))i + 2j + k$$

$$\nabla \times (\nabla \times \vec{A}) = 3i + 2j + k$$