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MECHATRONICS

16/ENG05/021

QUESTION 1

Mathematical modeling is the art of translating problems from an application area into tractable **mathematical** formulations whose theoretical and numerical analysis provides insight, answers, and guidance useful for the originating application.

Ii

Modelling and Simulation

Question 2

$r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t}K$

(i) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\, j + 6e^{2t}K$

(ii) $\frac{d^2r}{dt^2} = (2)i + 18\sin 3t\, j + 12e^{2t}K$

(iii) $\left|\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0)\, j + 12e^{2(0)}K$

$= 2i + 12K$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{(2)^2+(12)^2} = 12.17$

Question 3

$A = x^2y\,i + (xy+yz)\,j + xz^2K$

$B = yz\,i - 3xz\,j + 2xy\,K$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

(i) $\nabla\phi = \left(\frac{d}{dx} + \frac{d}{dy} + \frac{d}{dz}\right)(3x^2y + xyz - 4y^2z^2 - 3)$

$= (6xy+yz)i + (3x^2 + xz - 8yz^2)\,j + (xy - 8y^2z)K$

$= (6(1)(2) + (2)(1))i + (3(1)^2 + (1)(2) - 8(2)(1)^2)\,j + (1\times 2 - 8(2)^2(1))K$

$\nabla\phi = 14i - 12j - 30K$

(ii) $\nabla\cdot A = \left(\frac{d}{dx}i + \frac{d}{dy}j + \frac{d}{dz}K\right)(x^2y\,i + (xy+yz)\,j + xz^2K)$

$= (2xy\,i + y + z^2)\,i + (x^2 + (x+z))\,j + (y+2xz)K$

$= (2\times 1\times 2 + 2 + 1^2)\,i + (1^2 + (2))\,j + (2 + 2(1)(1))K$

$\nabla\cdot A = 6i + 3j + 4K$

(iii) $\nabla \times B = \begin{vmatrix} i & j & K \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$

$= \begin{vmatrix} d/dy & d/dz \\ -3xz & 2xy \end{vmatrix} i - \begin{vmatrix} d/dx & d/dz \\ yz & 2xy \end{vmatrix} j + \begin{vmatrix} d/dx & d/dy \\ yz & -3xz \end{vmatrix} K$

$= (2x + 3x)\,i - (2y - y)\,j + (-3z - z)\,K$

$= 5x\,i - y\,j - 4z\,K$

$\nabla \times B = 5(1)\,i - (2)\,j - 4x(1)\,K$

$\nabla \times B = 5i - 2j - 4K$

(iv) grad div A

$\text{div } A = 2xy\,i + (x+z)\,j + 2xz\,K$

$\text{grad div } A = \left(\frac{d}{dx}i + \frac{d}{dy}j + \frac{d}{dz}K\right)(2xy\,i + (x+z)\,j + 2xz\,K)$

$= (2y + (z+1) + 2z)\,i + (2x + 1)\,j + ((x+1) + 2x)\,K$

$= (2(2) + (1+1) + 2(1))\,i + (2(1))\,j + (2 + 2(1))\,K$

$\text{grad div } A = 8i + 2j + 4K$

(v) $\text{curl curl } A = \begin{vmatrix} i & j & K \\ d/dx & d/dy & d/dz \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$

$= \begin{vmatrix} d/dy & d/dz \\ (xy+yz) & xz^2 \end{vmatrix} i - \begin{vmatrix} d/dx & d/dz \\ x^2y & xz^2 \end{vmatrix} j + \begin{vmatrix} d/dx & d/dy \\ x^2y & (xy+yz) \end{vmatrix} K$

$\text{curl } A = (z^2 - y)\,i - (z^2)\,j + (y - x^2)\,K$

$\text{Curl curl } A = \begin{vmatrix} i & j & K \\ d/dx & d/dy & d/dz \\ (z^2-y) & -z^2 & (y-x^2) \end{vmatrix}$

$= \begin{vmatrix} d/dy & d/dz \\ -z^2 & (y-x^2) \end{vmatrix} i - \begin{vmatrix} d/dx & d/dz \\ (z^2-y) & (y-x^2) \end{vmatrix} j + \begin{vmatrix} d/dx & d/dy \\ (z^2-y) & -z^2 \end{vmatrix} K$

$= (1 - x^2 + 2z)\,i - (y - 2x - 2z - y)\,j + (z^2 - 1)\,K$

$\text{Curl (curl } A) = (i - 1^2 + 2(1))\,i - (2 - 2(1) - 2(1) - 2)\,j + (1^2 - 1)\,k$

$\text{Curl curl } A = 2i + 4j$