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 MAT No: 01EN02/011  
 DEPT: COMPUTER ENGINEERING

1) mathematically modelling is the process of setting up a model solving it mathematically and interpreting the result in physical or another term.

(ii) a) Using Balance law  $\rightarrow$  law of conservation of mass

b) Forming a differential equations from an existing algebraic equations of the system

2)  $r = (t^2 + 3t) i - 2 \sin 3t j + 3e^{2t} k$

(i)  $\frac{dr}{dt} = (2t + 3) i - 6 \cos 3t j + 6e^{2t} k$

(ii)  $\frac{d^2r}{dt^2} = \frac{d}{dt} \left( \frac{dr}{dt} \right)$   
 $= 2i + 18 \sin 3t j + 12e^{2t} k$

(iii)  $\left. \frac{d^2r}{dt^2} \right|_{t=0} = 2i + 18 \sin 3(0) j + 12e^{2(0)} k$   
 $= 2i + 12k$

3) if  $A = x^2 y i + (xy + yz) j + x^2 z k$

$B = yz i - 3xz j + 2xy k$  and

$\phi = 3x^2 y + xyz - 4y^2 z^2 - 3$

determine at point  $(1, 2, 1)$ .

i)  $\nabla \phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k$

$$\nabla\phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 4y^2z)k \text{ at point } (1, 2, 1)$$

$$\nabla\phi = (6(1)(2) + (2)(1))i + ((3(1)^2 + (1)(1)) - 8(2)(1)^2 + (1)(1) - 8(2)(1)^2)j + ((1)(2) - 4(2)2(1))k$$

$$\nabla\phi = 14i - 12j - 10k$$

$$ii) \nabla \cdot A = \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) \cdot axi + ayj + azk$$

$$= \frac{\partial ax}{\partial x} + \frac{\partial ay}{\partial y} + \frac{\partial az}{\partial z}$$

$$= \frac{\partial (x^2y)}{\partial x} + \frac{\partial (xy + yz)}{\partial y} + \frac{\partial (xz^2)}{\partial z}$$

$$= 2xy + (x + z) + 2cz$$

$$= 2(1)(2) + (1 + 1) + (1)2(1)$$

$$= 4 + 2 + 2$$

$$\text{at } (1, 2, 1)$$

$$\nabla \cdot A = 4 + 2 + 2$$

$$\nabla \cdot A = 8$$

$$iii) \nabla \times B = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -3xz & 2xy \end{vmatrix}$$

$$i \begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -3xz & 2xy \end{vmatrix} - j \begin{vmatrix} \frac{\partial}{\partial z} & \frac{\partial}{\partial x} \\ yz & 2xy \end{vmatrix} + k \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ yz & -3xz \end{vmatrix}$$

$$i \left[ \frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial z}(-3xz) \right] - j \left[ \frac{\partial}{\partial z}(2xy) - \frac{\partial}{\partial x}(yz) \right] + k \left[ \frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}(yz) \right]$$

$$\nabla \times B = i(2x + 3z) - j(2y - y) + k[-3z - z] \text{ at point } (1, 2, 1)$$

$$\nabla \times B = i(2(1) + 3(1)) - j(2(2) - (2)) + k(-3(1) - (1))$$

$$= 5i - 2j - 4k$$



iv grad div A

$$\text{div } A = \left( \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right) axi + byj + czk$$

$$= \frac{\partial ax}{\partial x} + \frac{\partial ay}{\partial y} + \frac{\partial az}{\partial z}$$

$$= \frac{\partial (2x^2y)}{\partial x} + \frac{\partial (xy + y^2)}{\partial y} + \frac{\partial xz^2}{\partial z}$$

$$= 2xy + (x+2) + xz^2$$

$$\text{grad div } A = \left( i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) \cdot (2xy + (x+2) + xz^2)$$

$$= (2y + z + 2z) i + j(2x) + (1+2) k$$

at point (1, 2, 1)

$$\text{grad div } A = (2(2) + 0 + 2(1)) i + (2(1)) j + (1+2) k$$

$$\nabla \cdot A = 7i + 2j + 3k$$

v curl curl A

$$\text{curl } A = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & (xy+y^2) & xz^2 \end{vmatrix}$$

$$i \left| \frac{\partial}{\partial y} \frac{\partial}{\partial z} \right| - j \left| \frac{\partial}{\partial x} \frac{\partial}{\partial z} \right| + k \left| \frac{\partial}{\partial x} \frac{\partial}{\partial y} \right|$$

$$\left| \frac{\partial}{\partial y} \frac{\partial}{\partial z} \right| = \begin{vmatrix} x^2y & xz^2 \\ xy+y^2 & xz^2 \end{vmatrix}$$

$$i [0 - x^2] - j [2^2 - 0] + k [y - x^2]$$

$$= -y i - 2^2 j + (y - x^2) k$$

$$+ k \left[ \frac{\partial}{\partial x} (-x^2) - \frac{\partial}{\partial y} (-y) \right]$$

$$\text{curl curl } A = i(1+2) - j(-2x+0) + k(0+1)$$

$$\text{curl curl } A = (1+2) i + 2x j + k$$

at point (1, 2, 1)

$$\text{curl curl } A = (1+2) i + 2(1) j + k$$

$$\text{curl curl } A = (1+2(1)) i + 2(1) j + k$$

$$= 3i + 2j + k$$

$$c) \left| \frac{d^2 r}{dt^2} \right|_{t=0}$$

$$\frac{d^2 r}{dt^2} = 2i + 18 \sin 30(0)j + 12e(0)$$

$$t=0 \quad 2i + 12k$$

$$\left| \frac{d^2 r}{dt^2} \right|_{t=0} = \sqrt{2^2 + 12^2}$$

$$= 12.165 \approx 12.17$$