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Mechanical Engineering
16/ENG06/028
ENG 282

ANSWERS TO ASSIGNMENT

1) Mathematical modeling can be defined as the process of setting up a model, solving it mathematically and interpreting the result in physical or other terms.

1(b) Two methods of obtaining models for engineering system are;

- Using the balance law (Law of conservation of mass).
- Forming a differential equation from an existing algebraic equation of the system.

2.(i) $r = (t^2+3t)i - (2\sin 3t)j + (3e^{2t})k$.

Recall that;

$$\frac{dr}{dt} = i\frac{dx(t)}{dt} + j\frac{dy(t)}{dy} + k\frac{dz(t)}{dy}$$

$$\frac{dr}{dt} = (2t+3)i - (6\cos 3t)j + (6e^{2t})k.$$

ii) $$\frac{d^2r}{dt^2} = 2i + (18\sin 3t)j + (12e^{2t})k.$$

iii) At $t=0$.

$$\frac{d^2r}{dt^2} = 2i + 0j + 12k.$$

$$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2+12^2} = 12.16$$

3) $A = x^2yi + (xy+yz)j + xz^2k$.
$B = yzi - 3xzj + 2xyk$.
$\phi = 3x^2y + xyz - 4y^2z^2 - 3$,

Determine at the point $(1,2,1)$

i) $\nabla\phi$

$$\nabla = \partial i/_{\partial x} + \partial j/_{\partial y} + \partial k/_{\partial z}.$$

$$\nabla\phi = \left(\partial i/_{\partial x} + \partial j/_{\partial y} + \partial k/_{\partial z}\right) \cdot \left(3x^2y + xyz - 4y^2z^2 - 3\right),$$

$$= (6xy + yz - 0 - 0)i + (3x^2 + xz - 8yz^2 - 0)j + (0 + xy - 8y^2z - 0)k,$$

$$\nabla\phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)k \quad \text{~~at point~~}$$

At point $(1,2,1)$.

$$\nabla\phi = (6(1)(2) + (2)(1))i + (3(1)^2 + (1)(1) - 8(2)(1)^2)j + ((1)(2) - 8(2)^2(1))k.$$

$$\nabla\phi = 14i - 12j - 30k.$$

$$\therefore \nabla\phi = 14i - 12j - 30k.$$

ii) $\nabla \cdot A$

$$\nabla \cdot A = \left(\partial i/_{\partial x} + \partial j/_{\partial y} + \partial k/_{\partial z}\right) \cdot (x^2yi + (xy+yz)j + xz^2k),$$

where;

$i \cdot i = j \cdot j = k \cdot k = 1$.

$$= \partial/_{\partial x}(x^2y) + \partial/_{\partial y}(xy+yz) + \partial/_{\partial z}(xz^2).$$

$$= 2xy + (x+z) + (2xz)k.$$

$$= 2xy + x + z + 2xz$$

$$= 2(1)(2) + (1+1) + 2(1)(1).$$

$$= 4 + 2 + 2.$$

$$\therefore \nabla \cdot A = 8.$$

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iii) $\overline{\nabla} \times B$

$$\overline{\nabla} \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$$= i\left(\partial/_{\partial y}(2xy) - \partial/_{\partial z}(-3xz)\right) - j\left(\partial/_{\partial x}(2xy) - \partial/_{\partial z}(yz)\right)$$
$$+ k\left(\partial/_{\partial x}(-3xz) - \partial/_{\partial y}(yz)\right).$$

$$= (2x + 3x)i - (2y - y)j + (-3z - z)k,$$
$$= 5xi - yj - 4zk.$$
$$= 5(1)i - 2j - 4(1)k.$$
$$\therefore \overline{\nabla} \times B = 5i - 2j - 4k.$$

iv) grad. div (A).

$$\nabla(\overline{\nabla}.A).$$

$$\nabla(\overline{\nabla}.A) = \left(\partial i/_{\partial x} + \partial j/_{\partial y} + \partial k/_{\partial z}\right)(2xy + xyz + 2xz)$$

From $(\overline{\nabla}.A) = 2xy + xyz + 2xz$ (calculation from II),

$$= \partial i/_{\partial x} + \partial j/_{\partial y} + \partial k/_{\partial z}(2xy + xyz + 2xz),$$

$$= 2y + (1+0) + (2z)i + (2y+0+0)j + (0+0+1+2x)k$$
$$= (2y + 1 + 2z)i + 2xj + (1 + 2x)k.$$
$$= 2(2) + 1(2(1))i + 2(1)j + (1 + 2(1)k).$$
$$= (4+3)i + 2j + 3k.$$
$$= 7i + 2j + 3k.$$
$$\therefore \nabla(\overline{\nabla}.A) = 7i + 2j + 3k$$

v) Curl curl A.

$$\text{curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= \left(\partial/_{\partial y}(xz^2) - \partial/_{\partial z}(xy+yz)\right)i - \left(\partial/_{\partial x}xz^2 - \partial/_{\partial z}x^2y\right)j$$
$$+ \left(\partial/_{\partial x}(xy+yz) - \partial/_{\partial y}(x^2y)\right)k$$

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$= i(0-y) - j(z^2-0) + k(y-x^2)$.

$\nabla \times A = -yi - z^2 j + (y-x^2)k$.

$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z & (y-x^2) \end{vmatrix}$

$= i\left[\frac{\partial}{\partial y}(y-x^2) - \frac{\partial}{\partial z}(-z^2)\right] - j\left[\frac{\partial}{\partial x}(y-x^2) - \frac{\partial}{\partial z}(-y)\right)$

$+ k\left[\frac{\partial}{\partial z}(-z^2) - \frac{\partial}{\partial y}(-y)\right)$

$= i(1+2z) - j(-2x+0) + k(0+1)$.

At $(1,2,1)$.

$\nabla \times (\nabla \times A) = i(1\times 2(1)) - j(-2(1)) + k(1)$.

$= 3i + 2j + k$.

$\therefore \nabla \times (\nabla \times A)$ at $(1,2,1) = 3i + 2j + k$.

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