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16/ENG01/015
Chemical Engineering

1) Mathematical modelling is the process of setting up a model solving it mathematically and interpreting the results in physical or in other terms

ii a) Using balance law - law of conversion of mass
b) Forming a differential equation from an existing algebraic equation of the system.

2) $r = (t^2 + 3t)i - 2\sin 3t\, j + 3e^{2t}k$

i) $\frac{dr}{dt} = (2t + 3)i - 6\cos 3t + 6e^{2t}k$

ii) $\frac{d^2r}{dt^2} = \frac{d}{dt}\left(\frac{dr}{dt}\right)$

$= 2i + 18\sin 3t\, j + 12e^{2t}k$

iii) $\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0) j + 12e^{2(0)}k$

$= 2i + 12k$

3) If

$A = x^2yi + (xy + yz)j + xz^2k$

$B = yzi - 3xzj + 2xyk$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

at point $(1,2,1)$

i) $\Delta\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}k$

$\Delta\phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 4y^2 2z$

at point $(1,2,1)$

$\Delta\phi = (6(1)(2) + (2)(1))i + ((3(1)^2 + (1)(1) - 8(2)(1)^2)j + ((1)(2) - 4(2)^2(1))k$

$\Delta\phi = 14i - 12j - 16k$

ii) $\Delta A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{dy} + \frac{\partial K}{\partial z}\right) \cdot a x i + a y j + a z k$

$= \frac{\partial a x}{\partial x} + \frac{\partial a y}{dy} + \frac{\partial a z}{\partial z}$

$= \frac{\partial x^2 y}{\partial x} + \frac{\partial (xy + yz)}{dy} + \frac{\partial}{\partial z}(x z^2)$

$= 2xy + (x+z) + x2z$

$= 2(1)(2) + (1+1) + (1)2(1)$

$= 4 + 2 + 2$

at $(1,2,1)$

$\Delta A = 4+2+2$

$\Delta A = 8$

iii) $\Delta \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3z & 2xy \end{vmatrix}$

$i \begin{vmatrix} \partial/dy & \partial/dz \\ -3xz & 2xy \end{vmatrix} - j \begin{vmatrix} \partial/dx & \partial/dz \\ yz & 2xy \end{vmatrix} + k \begin{vmatrix} \partial/dx & \partial/dy \\ yz & -3xz \end{vmatrix}$

$i\left[\partial/dy(2xy) - \partial/dz(-3xz)\right) - j\left[\partial/dx(2xy) - \partial/dz(yz)\right] + k\left[\partial/dx(-3xz) - \partial/dy(yz)\right]$

$\Delta \times B = i[2x + 3x] - j[2y - y] + k[-3z - z]$

at point $(1,2,1)$

$\Delta \times B = i(2(1) + 3(1)) - j[2(2) - (2)] + k[-3(1) - (1))$

$= 5i - 2j - 4k$

iv) grad div A

$\text{div } A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{dy} + \frac{\partial K}{\partial z}\right) \cdot a x i + a y j + a z k$

$= \frac{\partial a x}{\partial x} + \frac{\partial a y}{dy} + \frac{\partial a z}{\partial z}$

$= \frac{d(x^2 y)}{dx} + \frac{\partial}{dy}(xy + yz) + \frac{d x z^2}{dz}$

$= 2xy + (x+z) + 2xz = A$

$$\text{grad Div } A = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \right)(2xy + (x+z) + xz)$$

$$= (2y + 1 + z)i + j(2x) + (1+x)k$$

at point $(1,2,1)$

$$\text{grad Div } A = (2(2) + (1) + 1(1))i + (2(1))j + (1+1)k$$

$$\Delta\Delta A = 7i + 2j + 2k$$

v) Curl Curl A

$$\text{Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$i \begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy+y)z^2 & xz^2 \end{vmatrix} - j \begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2y & xz^2 \end{vmatrix} + k \begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2y & (xy+y^2) \end{vmatrix}$$

$$i[0 - y] - j[z^2 - 0] + k(y - x^2)$$

$$= -yi - z^2j + (y - x^2)k$$

$$+ k[\partial/\partial x(-z^2) - \partial/\partial y(-y)]$$

$$\text{Curl Curl } A = i[1 + 2z] - j[-2x + 0] + k[0 + 1]$$

$$\text{Curl Curl } A = (1 + 2z)i + 2xj + k$$

at point $(1,2,1)$

$$\text{Curl Curl } A = (1 + 2(1))i + 2(1)j + k$$

$$= 3i + 2j + k$$

(1) iii) $\left| \frac{d^2r}{dt^2} \right|$ at $t = 0$

$$\frac{d^2r}{dt^2} = 2i + 18\sin 30(0)j + 12e^{2(0)}$$

$t = 0$ $2i + 12k$

$$\left| \frac{d^2r}{dt^2} \right|_{t=0} = \sqrt{2^2 + 12^2}$$

$$= 12.17$$