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14. Eng 02/049 Computer Engineering
Assignment

② $r = (t^2 + 3t)i - 2 \sin 3t j + 3e^{2t} k$

i) $\frac{dr}{dt} = (2t+3)i - 6 \cos 3t j + 6e^{2t} k$

ii) $\frac{d^2r}{dt^2} = 2i + 18 \sin 3t j + 12e^{2t} k$

iii) $\frac{d^2r}{dt^2} = 2i + 12k$

$$\left| \frac{d^2r}{dt^2} \right| = \sqrt{2^2 + 12^2} = \sqrt{4 + 144} \\ = \sqrt{148} = 2\sqrt{37} \\ \approx 12.7$$

③ $A = x^2 i + (xy + y^2) j + xz^2 k$

$B = yz i - 3xz^2 j + 2xy k$

$\phi = 3x^2 y + xz^2 - 4y^2 z^2 - 3$

i) $\nabla \phi = \frac{\partial \phi}{\partial x} i + \frac{\partial \phi}{\partial y} j + \frac{\partial \phi}{\partial z} k$

$\frac{\partial \phi}{\partial x} = 6xy + z^2$

$\frac{\partial \phi}{\partial y} = xz - 8yz^2$

$\frac{\partial \phi}{\partial z} = 2xz^2 + xz - 8yz^2$

At $(1, 2, 1)$

$\frac{\partial \phi}{\partial x} = 6(1)(2) + (1) = 12 + 1 = 13$

$\frac{\partial \phi}{\partial y} = 3(1) + 1(1) - 8(2)(1)^2 = 3 + 1 - 16 = -12$

$\frac{\partial \phi}{\partial z} = (1)(2) - 8(2)^2(1) = 2 - 32 = -30$

$$\nabla \cdot A = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$$

$$A = ax^2 + ay^2 + az^2$$

$$\nabla \cdot A = 2ax + (x+1) + 2xz$$

$$At (1, 2, 1)$$

$$\nabla \cdot A = 2(1)(2) + (1+1) + 2(1)(1)$$

$$= 4+2+2 = 8$$

$$\nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ J_x & -3xz & 2xz \end{vmatrix}$$

$$= \hat{i}(2x+3xz) - \hat{j}(2z-3) + \hat{k}(-3z-2)$$

$$= 5x\hat{i} - 3\hat{j} - 4\hat{k}$$

$$At (1, 2, 1)$$

$$\nabla \times B = 5\hat{i} - 3\hat{j} - 4\hat{k}$$

grad div A

$$\text{grad}(2xy + (x+2) + 2xz)$$

$$(\rho + \text{div} A = C = \nabla A$$

$$\nabla(\nabla A) = \nabla C = \hat{i} \frac{\partial C}{\partial x} + \hat{j} \frac{\partial C}{\partial y} + \hat{k} \frac{\partial C}{\partial z}$$

$$= \hat{i}(2y+1+2z) + \hat{j}(2x) + \hat{k}(1+2x)$$

at (1, 2, 1)

$$\nabla C = \left[\hat{i}(2(2)+1+2(1)) + \hat{j} 2(1) + \hat{k}(1+2(1)) \right]$$

$$= 7\hat{i} + 2\hat{j} + 3\hat{k}$$

curl A

$$\text{curl} A = \nabla \times A$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ x^2 & (xy+y^2) & xz^2 \end{vmatrix}$$

$$= \hat{i}(0-y) - \hat{j}(z^2-0) + \hat{k}(y-x^2)$$

$$= -y\hat{i} - z^2\hat{j} + \hat{k}(y-x^2)$$

at (1, 2, 1)

$$\text{curl} = -2\hat{i} - \hat{j} + \hat{k}$$

$$\text{Curl } A = \nabla \times (\nabla \times A)$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -1 & -2z & 1-x^2 \end{vmatrix}$$

$$= \hat{i}(1+2z) - \hat{j}(-2x-0) + \hat{k}(0+1)$$

$$= \hat{i}(1+2z) + 2x\hat{j} + \hat{k}$$

at point (1, 2, 1)

$$\nabla \times (\nabla \times A) = \hat{i}(1+2(1)) + 2(1)\hat{j} + \hat{k}$$

$$= 3\hat{i} + 2\hat{j} + \hat{k}$$

1a. A mathematical model is a description of a system using mathematical concepts and language. Therefore modelling is the process of setting up a model, solving it mathematically and interpreting the result in physical and other terms.

b. Exponential Growth Decay (use of ODE)

i. Mixing Problems