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Solution

Question 1:

i) Mathematical modelling is the process of setting up ~~the~~ a model for an engineering problem, solving it mathematically and interpret the results in physical or other terms.

ii) → Mathematical
→ Physical

Question 2

$$r = (t^2+3t)i - (2\sin 3t)j + (3e^{2t})k$$

i) $\frac{dr}{dt}$

$$\frac{dr}{dt} = (2t+3)i - (6\cos 3t)j + (6e^{2t})k$$

ii) $\frac{d^2r}{dt^2}$

$$\frac{d^2r}{dt^2} = 2i - (-6 \cdot 3\sin 3t)j + (6 \cdot 2e^{2t})k$$

$$\frac{d^2r}{dt^2} = 2i + (18\sin 3t)j + 12e^{2t}k$$

iii) $$\left.\frac{d^2r}{dt^2}\right|_{t=0} = 2i + 18\sin 3(0)j + 12e^{2(0)}k$$

$$\therefore \left.\frac{d^2r}{dt^2}\right| = 2i + 12k$$

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Question 3:

$A = x^2y\,i + (xy + yz)\,j + xz^2\,k$

$B = yz\,i - 3xz\,j + 2xy\,k$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

at $(1,2,1)$.

(i) $\nabla\phi$

$$\nabla\phi = \left(i\frac{\delta}{\delta x} + j\frac{\delta}{\delta y} + k\frac{\delta}{\delta z}\right)(3x^2y + xyz - 4y^2z^2 - 3)$$

$$\nabla\phi = i\frac{\delta}{\delta x}(3x^2y + xyz - 4y^2z^2 - 3) + j\frac{\delta}{\delta y}(3x^2y + xyz - 4y^2z^2 - 3) \cancel{+k\delta}$$
$$+ k\frac{\delta}{\delta z}(3x^2y + xyz - 4y^2z^2 - 3)$$

$$\nabla\phi = i(6xy + yz) + j(3x^2 + xz - 8yz^2) + k(xy - 8y^2z)$$

$$\nabla\phi\Big|_{(1,2,1)} = [6(1)(2) + (2)(1)]i + [3(1)^2 + (1)(1) - 8(2)(1)^2]j + [(1)(2) - 8(1)^2(1)]k$$

$$\nabla\phi\Big|_{(1,2,1)} = 14i \cancel{+} -12j \cancel{+} -30k$$

$$\therefore \nabla\phi\Big|_{(1,2,1)} = 14i - 12j - 30k$$

(ii) $\nabla\cdot A$

$$\nabla\cdot A = \left(i\frac{\delta}{\delta x} + j\frac{\delta}{\delta y} + k\frac{\delta}{\delta z}\right)(x^2y\,i + (xy + yz)\,j + xz^2\,k)$$

$$\nabla\cdot A = \frac{\delta}{\delta x}(x^2y)\,i\cdot i + \frac{\delta}{\delta y}(xy + yz)\,j\cdot j + \frac{\delta}{\delta z}(xz^2)\,k\cdot k$$

$$\nabla\cdot A = \frac{\delta}{\delta x}(x^2y) + \frac{\delta}{\delta y}(xy + yz) + \frac{\delta}{\delta z}(xz^2)$$

$$\nabla\cdot A = 2xy + (x + z) + 2xz$$

$$\nabla\cdot A\Big|_{(1,2,1)} = 2(1)(2) + (1+1) + 2(1)(1)$$

$$\nabla\cdot A\Big|_{(1,2,1)} = 4 + 2 + 2 = 8$$

$$\nabla\cdot A\Big|_{(1,2,1)} = 8$$

iii) $\nabla\times B$

$$\nabla\times B = \begin{vmatrix} i & j & k \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ yz & -3xz & 2xy \end{vmatrix}$$

$$\nabla\times B = i\begin{vmatrix} \delta/\delta y & \delta/\delta z \\ -3xz & 2xy \end{vmatrix} - j\begin{vmatrix} \delta/\delta x & \delta/\delta z \\ yz & 2xy \end{vmatrix} + k\begin{vmatrix} \delta/\delta x & \delta/\delta y \\ yz & -3xz \end{vmatrix}$$

$$\nabla\times B = i\left(\frac{\delta}{\delta y}(2xy) + \frac{\delta}{\delta z}(3xz)\right) - j\left(\frac{\delta}{\delta x}(2xy) - \frac{\delta}{\delta z}(yz)\right) + k\left(\frac{\delta}{\delta x}(-3xz) - \frac{\delta}{\delta y}(yz)\right)$$

$$\nabla\times B = i(2x + 3x) - j(2y - y) + k(-3z - z)$$

$$\nabla\times B = (5x)i - j(y) - (4z)k$$

$$\nabla\times B\Big|_{(1,2,1)} = 5(1)i - (2)j - 4(1)k$$

$$\nabla\times B\Big|_{(1,2,1)} = 5i - 2j - 4k$$

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iv) grad div A

$\text{div } A = \nabla \cdot A$

and from the previously solved (ii)

$$\nabla \cdot A = 2xy + (x+z) + 2xz$$

$$\therefore \text{grad div } A = \nabla(\nabla \cdot A)$$

$$\text{grad div } A = \left(i\frac{\delta}{\delta x} + j\frac{\delta}{\delta y} + k\frac{\delta}{\delta z}\right)(2xy + x + z + 2xz)$$

$$\text{grad div } A = i\frac{\delta}{\delta x}(2xy + x + z + 2xz) + j\frac{\delta}{\delta y}(2xy + x + z + 2xz) + k\frac{\delta}{\delta z}(2xy + x + z + 2xz)$$

$$\text{grad div } A = i(2y + 1 + 2z) + j(2x) + k(1 + 2x)$$

$$\text{grad div } A \Big|_{(1,2,1)} = (2(2) + 1 + 2(1))i + (2(1))j + (1 + 2(1))k$$

$$\text{grad div } A \Big|_{(1,2,1)} = 7i + 2j + 3k$$

(v) Curl Curl A =

Curl A = $\nabla \times A$

$$\nabla \times A = \begin{vmatrix} i & j & k \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ x^2y & (xy+yz) & (xz^2) \end{vmatrix}$$

$$\nabla \times A = i \begin{vmatrix} \delta/\delta y & \delta/\delta z \\ (xy+yz) & xz^2 \end{vmatrix} - j \begin{vmatrix} \delta/\delta x & \delta/\delta z \\ x^2y & xz^2 \end{vmatrix} + k \begin{vmatrix} \delta/\delta x & \delta/\delta y \\ x^2y & (xy+yz) \end{vmatrix}$$

$$\nabla \times A = i\left(\frac{\delta}{\delta y}(xz^2) - \frac{\delta}{\delta z}(xy+yz)\right) - j\left(\frac{\delta}{\delta x}(xz^2) - \frac{\delta}{\delta z}(x^2y)\right) + k\left(\frac{\delta}{\delta x}(xy+yz) + - \frac{\delta}{\delta y}(x^2y)\right)$$

$$\nabla \times A = i(0 - y) - j(z^2 - 0) + k((y+z) - x^2)$$

$$\nabla \times A = (-y)i - (z^2)j + (\cancel{x}\, y+z-x^2)k$$

Curl Curl A = $\nabla \times (\nabla \times A)$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \delta/\delta x & \delta/\delta y & \delta/\delta z \\ -y & -z^2 & (y+z-x^2) \end{vmatrix}$$

$$\nabla \times (\nabla \times A) = i \begin{vmatrix} \delta/\delta y & \delta/\delta z \\ -z^2 & (y+z-x^2) \end{vmatrix} - j \begin{vmatrix} \delta/\delta x & \delta/\delta z \\ -y & (y+z-x^2) \end{vmatrix} + k \begin{vmatrix} \delta/\delta x & \delta/\delta y \\ -y & -z^2 \end{vmatrix}$$

$$\nabla \times (\nabla \times A) = i\left(\frac{\delta}{\delta y}(y+z-x^2) + \frac{\delta}{\delta z}(z^2)\right) - j\left(\frac{\delta}{\delta x}(y+z-x^2) + \frac{\delta}{\delta z}(y)\right) + k\left(\frac{\delta}{\delta x}(-z^2) + \frac{\delta}{\delta y}(y)\right)$$

$$\nabla \times (\nabla \times A) = i(1 + 2z) - j(-2x + 0) + k(0 + 1)$$

$\nabla \times (\nabla \times A) = (1+2z)i + (2x)j + k$

$\nabla \times (\nabla \times A)\Big|_{(1,2,1)} = (1+2(1))i + (2(1))j + k$

$\nabla \times (\nabla \times A)\Big|_{(1,2,1)} = 3i + 2j + k$