16/ENG04/019

NAME: FASHINA, Oluwadamilola .T.
MATRIC NO: 16/ENG04/019
DEPT: Electrical/Electronics Engineering

Engineering Mathematics Assignment

Question 1

1) Define mathematical Modelling.

Modelling is a process of setting up a model, solving it mathematically, and interpreting the result in physical or other terms is called mathematical Modelling

2) Outline two methods of obtaining models for engineering systems

1. By differentiation.
2. By the rules of law of conservation of mass

Question 2.

If $r = (t^2 + 3t)i - 2\sin 3t\,j + 3e^{2t}k$

Determine

i) $\frac{dr}{dt}$

ii) $\frac{d^2r}{dt^2}$

iii) the value of $\left|\frac{d^2r}{dt^2}\right|$, $t = 0$.

Solution.

a) $\frac{dr}{dt} = (2t + 3)i - 6\cos 3t\,j + 6e^{2t}k$

b) $\frac{d^2r}{dt^2} = 2i + 18\sin 3t\,j + 12e^{2t}k.$

$= (2y + 1 + 2z)i + 2xj + (1+2x)k$

$= 2(2) + 1(2(1))\ i + 2(1)j + (1+2(1))k)$

$= (4+3)i + 2j + 3k$

$= 7i + 2j + 3k.$

$\therefore \bar{\nabla}(\bar{\nabla}\cdot A) = 7i + 2j + 3k.$

(v) Curl curl A

$$\text{Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy+yz)\right)i - \left(\frac{\partial}{\partial x} xz^2 - \frac{\partial}{\partial z} x^2y\right)j$$

$$+ \left(\frac{\partial}{\partial x}(xy+yz) - \frac{\partial}{\partial y}(x^2y)\right)k.$$

$= i(0-y) - j(z^2 - 0) + k(y - x^2)$

$\bar{\nabla}\times A = -yi - z^2j + (y - x^2)k.$

$$\bar{\nabla}\times(\bar{\nabla}\times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= i\left(\frac{\partial}{\partial y}(y-x^2) - \frac{\partial}{\partial z}(-z^2)\right) - j\left(\frac{\partial}{\partial x}(y-x^2) - \frac{\partial}{\partial z}(-y)\right)$$

$$+ k\left(\frac{\partial}{\partial z}(-z^2) - \frac{\partial}{\partial y}(-y)\right)$$

$= i(1+2z) - j(-2x+0) + k(0+1)$

$\bar{\nabla}\times(\bar{\nabla}\times A)$ at $(1,2,1)$.

$\bar{\nabla}\times(\bar{\nabla}\times A) = i(1\times 2(1)) - j(-2(1)) + k(1).$

$= 3i + 2j + k$

$\therefore \bar{\nabla}\times(\bar{\nabla}\times A)$ at $(1,2,1) = 3i + 2j + k$

(ii) $\nabla \cdot A$

$\nabla \cdot A = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot \left(x^2 y i + (xy + yz) j + xz^2 k\right)$

where $i \cdot i = j \cdot j = k \cdot k = 1$

$= \frac{\partial}{\partial x}(x^2 y) + \frac{\partial}{\partial y}(xy + yz) + \frac{\partial}{\partial z}(xz^2)$

$= 2xy + (x + z) + (2xz)$

$= 2xy + x + z + 2xz$

$= 2(1)(2) + (1 + 1) + 2(1)(1)$

$= 4 + 2 + 2$

$\nabla \cdot A \quad \underline{= 8}$

(iii) $\nabla \times B$

$$\nabla \times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$= i\left(\frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial z}(-3xz)\right) - j\left(\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}(yz)\right)$

$+ k\left(\frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}(yz)\right)$.

$= (2x + 3x) i - (2y - y) j + (-3z - z) k$

$= 5xi - yj - 4zk$

$= 5(1)i - 2j - 4(1)k$

$\nabla \times B \quad = \underline{5i - 2j - 4k}$

(iv) grad. div(A)

$\nabla(\nabla \cdot A)$

$\nabla(\nabla \cdot A) = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right)(2xy + xyz + 2xz)$

from $(\nabla \cdot A) = 2xy + xyz + 2xz$ (Calculation from ii)

$= \frac{\partial i}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial k}{\partial z}(2xy + xyz + 2xz)$

$= 2y + (1 + 0) + (2z) i + (2y + 0 + 0) j + (0 + 0 + 1 + 2x) k$

(iv) $\left|\frac{d^2r}{dt^2}\right|$ at $t=0$.

at $t=0$

$= 2i + 18\sin 3(0)j + 12e^{2(0)}k$

$= 2i + 0j + 12k$

$= 2i + 12k$

$\left|\frac{d^2r}{dt^2}\right|$ at $t=0$

$= \sqrt{(2i)^2 + (12k)^2}$

$= \sqrt{4 + 144}$

$= \sqrt{148}$

$= 12.166$

$= 12.17$

Question 3

$A = x^2yi + (xy+yz)j + xz^2k$

$B = yzi - 3xzj + 2xyk$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

determine at the point $(1, 2, 1)$

(i) $\nabla\phi$

$\nabla = \frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}$

$\nabla\phi = \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot (3x^2y + xyz - 4y^2z^2 - 3)$

$= (6xy + yz - 0 - 0)i + (3x^2 + xz - 8yz^2 - 0)j + (0 + xy - 8y^2z - 0)k$

$\nabla\phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)k$

at point $(1, 2, 1)$.

$\nabla\phi = (6(1)(2) + (2)(1))i + (3(1)^2 + (1)(1) - 8(2)(1)^2)j + ((1)(2) - 8(2)^2(1))k$

$\nabla\phi = 14i - 12j - 30k$

$\nabla\phi = 14 - 12j - 30k$