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16/ENG03/041.

ASSIGNMENT III
CIVIL ENGINEERING

(1) Mathematical modelling:
It is the process of setting up a model, Solving it mathematically, and interpreting the result in physical or other terms.

(11) 2 methods of obtaining models:
- Numeric Method by Euler
- Extended Method by reduction to Seperable.

(2) $r = (t^2-3t)\hat{i} - 2\sin 3t\hat{j} + 3e^{2t}k$

$\frac{dr}{dt} = (2t-3)\hat{i} - 6\cos 3t\hat{j} + 6e^{2t}\hat{k}$

$\frac{d^2r}{dt^2} = 2\hat{i} + 18\sin 3t\hat{j} + 12e^{2t}\hat{k}$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = 2\hat{i} + 12\hat{k}$

$= \sqrt{2^2 + 12^2} = 2\sqrt{37}$

$\therefore \left|\frac{d^2r}{dt^2}\right|_{t=0} = 12.2$

(3) $A = x^2y\hat{i} + (xy+yz)\hat{j} + xz^2\hat{k}$

$B = yz\hat{i} - 3xz\hat{j} + 2xy\hat{k}$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$.

at point $(1, 2, 1)$.

(i) $\nabla\phi$

$\text{grad}\,\phi = \frac{d\phi}{dx}\hat{i} + \frac{d\phi}{dy}\hat{j} + \frac{d\phi}{dz}\hat{k}$.

$\frac{d\phi}{dx}\hat{i} = (6xy + yz)\hat{i} = 14\hat{i}$

$\frac{d\phi}{dy}\hat{j} = 3x^2 + xz - 8yz^2\hat{j}$

$= -12\hat{j}$

$\frac{d\phi}{dz}\hat{k} = xy - 8y^2z = -30\hat{k}$

$\therefore \nabla\phi = 14\hat{i} - 12\hat{j} - 30\hat{k}$.

(ii) $\nabla \cdot A$

$\nabla \cdot A = \frac{da_x}{dx} + \frac{da_y}{dy} + \frac{da_z}{dz}$

$\frac{da_x}{dx} = 2xy = 4$.

$\frac{da_y}{dy} = x + z = 2$

$\frac{da_z}{dz} = 2xz = 2$

$\therefore \nabla \cdot A = 8$.

(iii) $\nabla \times B. = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ yz & -3xz & 2xy \end{vmatrix}$

$= \hat{i}\left(\frac{d(2xy)}{dy} - \frac{d(-3xz)}{dz}\right) - \hat{j}\left(\frac{d(2xy)}{dx} - \frac{d(yz)}{dz}\right)$

$+ \hat{k}\left(\frac{d(-3xz)}{dx} - \frac{d}{dy}(yz)\right)$.

$= \hat{i}(2x + 3x) - \hat{j}(2y - y) + \hat{k}(-3z - z)$.

$= \hat{i}(2x + 3x) - \hat{j}(2y - y) - \hat{k}(3z + z)$.

$= 5\hat{i} - 2\hat{j} - 4\hat{k}$.

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(iv) grad div A =

grad$(\nabla\cdot A)$.

$\nabla\cdot A = 2xy + (x+z) + 2xz$

$\therefore \text{grad}(\nabla\cdot A) = \frac{d(\nabla\cdot A)}{dx}\hat{i} + \frac{d(\nabla\cdot A)}{dy}\hat{j} + \frac{d(\nabla\cdot A)}{dz}\hat{k}$

$\frac{d(\nabla\cdot A)}{dx}\hat{i} = (2y + 1 + 2z)\hat{i}$.

$\frac{d(\nabla\cdot A)}{dy}\hat{j} = 2x\hat{j}$.

$\frac{d(\nabla\cdot A)}{dz}\hat{k} = (1 + 2x)\hat{k}$.

$\therefore$ grad div A $= (2y+1+2z)\hat{i} + 2x\hat{j} + (1+2x)\hat{k}$

grad div A $= 7\hat{i} + 2\hat{j} + 3\hat{k}$

(v) Curl Curl A.

$= \nabla\times(\nabla\times A)$.

$$\nabla\times A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$$= \hat{i}\left(\frac{d(xz^2)}{dy} - \frac{d(xy+yz)}{dz}\right) - \hat{j}\left(\frac{d(xz^2)}{dx} - \frac{d(x^2y)}{dz}\right) + k\left(\frac{d(xy+yz)}{dx} - \frac{d(x^2y)}{dy}\right).$$

$= \hat{i}(-y) - \hat{j}(z^2) + \hat{k}(y - x^2)$.

$$\nabla\times(\nabla\times A) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ d/dx & d/dy & d/dz \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$$= \hat{i}\left(\frac{d(y-x^2)}{dy} - \frac{d(-z^2)}{dz}\right) - \hat{j}\left(\frac{d(y-x^2)}{dx} - \frac{d(-y)}{dz}\right) + k\left(\frac{d(-z^2)}{dx} - \frac{d(-y)}{dy}\right).$$

$= \hat{i}(1+2z) - \hat{j}(-2x) + k(1)$.

$= 3\hat{i} + 2\hat{j} + \hat{k}$.

Curl Curl A $= 3\hat{i} + 2\hat{j} + \hat{k}$.