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MAT. NO: 17/ENG05/004
DEPT: MECHATRONICS
COURSE: ENG282 Assignment 3

1 Mathematical Modeling is defined as the process of setting up a model, solving it mathematically and interpreting the result in physical or other terms

b) Using the balance law (Law of ~~conduction~~ conservation of mass)
Use of Mixing Problem equation

2 $r = (t^2 + 3t)i - 2\sin 3t\,j + 3e^{2t}k$

i) $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\,j + 6e^{2t}k$

$\frac{d^2r}{dt^2} = 2i + 18\cos 3t\,j + 12e^{2t}k$

$\frac{d^2r}{dt^2}$ at $t = 0$

$= 2i + 18\sin 3(0)j + 12e^{2(0)}k$

$= 2i + 0j + 12k$

$= 2i + 12k$

3 $\nabla\phi = \left(\frac{\partial\phi}{\partial x}\right)i + \left(\frac{\partial\phi}{\partial y}\right)j + \left(\frac{\partial\phi}{\partial z}\right)k$

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$ (

$\nabla\phi = \frac{(3x^2y + xyz - 4y^2z^2 - 3)}{dx}i + \frac{(3x^2y + xyz - 4y^2z^2 - 3)}{dy}j$

$+ \frac{(3x^2y + xyz - 4y^2z^2 - 3)}{dz}k$

$\nabla\phi = (6xy + yz)i + (3x^2 + xz - 8yz^2)j + (xy - 8y^2z)k$

at $(x, y, z) = (1, 2, 1)$

$\nabla\phi = 6(1\times2) + (2\times1)i + (3(1)^2 + (1\times1) - 8(2\times1^2)j + (1\times2 - 8\times2^2\times1)k$

$= 14i - 12j - 30k$

ii) $\text{div } A = \nabla \cdot A$

$= \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\right) \cdot \left(x^2 y i + (xy + yz) j + xz^2 k\right)$

$= \frac{\partial (x^2 y)}{\partial x} i + \frac{\partial (xy + yz)}{\partial y} j + \frac{\partial (xz^2)}{\partial z} k$

$= (2xy) i + (x + z) j + (2xz) k$ at $(1, 2, 1)$

$= 2(1 \times 2) + (1 + 1) + (2 \times 1 \times 1) = 8$

iii) $\nabla \times B = \text{Curl } B$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (yz) & (-3xz) & (2xy) \end{vmatrix}$$

$= \left(\frac{\partial (2xy)}{\partial y} - \frac{\partial (-3xz)}{\partial z}\right) i - \left(\frac{\partial (2xy)}{\partial x} - \frac{\partial (yz)}{\partial z}\right) j + \left(\frac{\partial (-3xz)}{\partial x} - \frac{\partial (yz)}{\partial y}\right) k$

$= (2x + 3x) i - (2y - y) j + k(-3z - z)$

$= 5x i - y j - 4z k$

$= 5(1) i - (2) j - 4(1) k$ at $(1, 2, 1)$

$= 5i - 2j - 4k$

Grad div $A = \nabla(\nabla \cdot A)$

$\nabla \cdot A = 2xy + x + z + 2xz$ from question (2)

$\nabla(\nabla \cdot A) = \frac{\partial (2xy + x + z + 2xz)}{\partial x} i + \frac{\partial (2xy + x + z + 2xz)}{\partial y} j + \frac{\partial (2xy + x + z + 2xz)}{\partial z} k$

$= (2y + 1 + 2z) i + (2x) j + (1 + 2x) k$

$= [2(2) + 1 + 2(1)] i + 2(1) j + [1 + 2(1)] k$

$= 7i + 2j + 3k$

iv Curl Curl A

$A = (x^2y)i + (xy+y^2)j + xz^2k$

$$\text{Curl } A = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & xy+yz & xz^2 \end{vmatrix}$$

$$= i\left(\frac{\partial(xz^2)}{\partial y} - \frac{\partial(xy+yz)}{\partial z}\right) - j\left(\frac{\partial(xz^2)}{\partial x} - \frac{\partial(x^2y)}{\partial z}\right) + k\left(\frac{\partial(xy+yz)}{\partial x} - \frac{\partial(x^2y)}{\partial y}\right)$$

$$= i(0-y) - j(z^2-0) + k(y-x^2)$$

$$\text{Curl } A = \bar{\nabla} \times A = -yi - z^2j + (y-x^2)k$$

$$\bar{\nabla} \times (\bar{\nabla} \times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & y-x^2 \end{vmatrix}$$

$$= i\left[\frac{\partial(y-x^2)}{\partial y} - \frac{\partial(-z^2)}{\partial z}\right] - j\left(\frac{\partial(y-x^2)}{\partial x} - \frac{\partial}{\partial z}(-y)\right) + k\left[\frac{\partial(-z^2)}{\partial z} - \frac{\partial}{\partial y}(-y)\right)$$

$$= i(1+2z) - j(-2x+0) + k(0+1)$$

At $(1, 2, 1)$

$$(\bar{\nabla} \times A) = i(1 \times 2(1)) - j(-2(1)) + k(1)$$

$$= 3i + 2j + k.$$

$$\text{Curl Curl } A = \bar{\nabla} \times (\bar{\nabla} \times A) = i(1 \times 2(1)) - j(-2(1)) + k(1)$$

$$= 3i + 2j + k$$