OKoduwa Odion James
Petroleum Engineering
16/ENG 07/020
ENG(282)

1. A mathematical metal is a description of a System using mathematical Concepts and Language. Therefore, modelling is the process of Setting up a model, Solving it mathematically and interpreting the result in physical and clear terms.

b Exponential growth/decay (use of ODE)
ii) Mixing problems

2 $v = (t^3 + 3t)i - 2\sin 3t\hat{j} + 3e^{2t}\hat{k}$

i $\frac{dr}{dt} = (2t+3)i - 6\cos 3t\hat{j} + 6e^{2t}\hat{k}$

ii $\frac{d^3r}{dt^2} = 2\hat{i} + 18\sin 3t\hat{j} + 12e^{2t}\hat{k}$

iii $\left.\frac{d^3r}{dt^2}\right|_{t=0} = 2\hat{i} + 12\hat{k}$

$\left|\frac{d^2r}{dt^2}\right| = \sqrt{2^2+12^2} = \sqrt{148} = 2\sqrt{37}$
$= 12.17$

3) $A = x^2yi + (xy+yz)\hat{j} + z^2\hat{k}$
$B = yzi - 3xz\hat{j} + 2xy\hat{k}$
$\phi = 3x^2y + xyz - Ky^2z^2 - 3$

i) $\nabla\phi = \frac{\partial\phi}{\partial x} i + \frac{\partial\phi}{dy} j + \frac{\partial\phi}{dz}\hat{k}$

$\frac{d\phi}{dx} = 6xy + yz$; $\frac{d\phi}{dy} = 3x^2 + xz - 8yz^2$

$\frac{d\phi}{dz} = xy - 8y^2z$

At $(\overset{x}{1}, \overset{y}{2}, \overset{z}{1})$

$\frac{d\phi}{dx} = 6(1)(2) + (2)(1) = 12 + 2 = 14$

$\frac{d\phi}{dx} = 3(1) + (1)(1) - 3(2)\cdot(1)^2 = 12$

$\frac{d\phi}{dz} = (1)\cdot(2) - 8(2)^2(1) = 2 - 32 = -30$

$\nabla\cdot\phi = 14i + 12\hat{j} - 30\hat{k}$

ii $\nabla\cdot A = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$\cancel{\nabla}\cdot A = a_x i + a_y \hat{j} + a_z k$

$\nabla\cdot A = 2xy + (x+z) + 2xz$

At $(1\ 2\ 1)$

$\nabla\cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$= 4 + 2 + 2 = 8$

iii) $\nabla\times B$

$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ \partial/dx & d/dy & \partial/dz \\ yz & -3xz & 2xy \end{vmatrix}$$

$= i(2x + 3x) - j(2y - y) + \hat{k}(-3z - z)$

$= 5x\hat{i} - y\hat{j} - 4z k$

$-8yz^2$

At $\begin{pmatrix} x & y & z \\ 1 & 2 & 1 \end{pmatrix}$

$\nabla \times B = 5i - 2j - 4k$

grad div A

grad $(2xy + (x+z) + 2xz)$

let div $A = C = \nabla A$

$\nabla(\nabla A) = \nabla C = i\frac{dc}{dx} + \hat{j}\frac{dc}{dy} + \hat{k}\frac{dc}{dz}$

$i(2y + 1 + 2z) + j(2x) + a(1 + 2k)$

At $(1\ 2\ 1)$

$\nabla C = i(2(2) + 1 + 2(1)) + j(2(1)) + \hat{k}(1 + (2)(1))$

$i(4 + 1 + 2) + j(2) + k(1+2)$

$= 7\hat{i} + 2j + 3k$

Curl curl A

Curl $A = \nabla \times A$

$$= \begin{vmatrix} i & j & k \\ \partial/dx & \partial/dy & \partial/dz \\ x^2y & (xy+yz) & xz^2 \end{vmatrix}$$

$= \hat{i}(0 - y) - \hat{j}(z^2 - 0) + \hat{k}(y - x^2)$

$= -y\hat{i} - z^2\hat{j} + k\hat{}(y - k^2)$

At $(1\ 2\ 1)$

Curl $A = -2\hat{i} - \hat{j} + k$

Curl curl $A = \nabla \times (\nabla \times A)$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \partial/dx & \partial/dy & \partial/dz \\ -y & -z^2 & (y-x^2) \end{vmatrix}$$

$= \hat{i}(1 + 2z - \hat{j}(-2x^2 - 0) + \hat{k}(0+1)$

$= \hat{j}(1 + 2z) + 2x^2\hat{j} + \hat{k}$

At point (1, 2, 1)

$$\nabla \times (\nabla \times A) = \hat{i}\,(1+2\cdot(1)) + 2(1)^2\,\hat{j} + \hat{k}$$

$$= 3\hat{i} + 2\hat{j} + \hat{k}$$