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16/ENG06/004
MECHANICAL ENGINEERING
200 Level

ENG 282

a Modelling can be defined as the process of setting up a model, solving it mathematically and ~~set~~ interpreting the result in physical or other forms

METHODS OF CREATING MODELS

i) ~~Radioactivity, exponential growth/decay~~
ii) ~~Mixing problems~~ Differentiating a linear equation
ii) By using Balance law

2 $r = (t^2 + 8t)\hat{i} - 2\sin 3t\hat{j} + 3e$

i) $\frac{dr}{dt} = (2t + 3)\hat{i} - 6\cos 3t\hat{j} + 6e^{2t}\hat{k}$

ii) $\frac{d^2r}{dt^2} = 2\hat{i} + 18\sin 3t\hat{j} + 12e^{2t}\hat{k}$

iii) $\frac{d^2r}{dt^2}$ at $t = 0$

$= 2\hat{i} + 18\sin 0\hat{j} + 12e^{2\times 0}\hat{k}$

$\frac{d^2r}{dt^2} = 2\hat{i} + 12\hat{k}$

3) $A = x^2y\hat{i} + (xy + yz)\hat{j} + xz^2\hat{k}$

$B = yz\hat{i} + 3xz\hat{j} + 2xy\hat{k}$

$\phi = 3x^2y + xyz - 4y^2z^3 - 3$

at point $(1, 2, 1)$

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i) $\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$

$\nabla\phi = (6xy + yz)\hat{i} - (3x^2 + xz - 8yz^2)\hat{j} + (xy - 8y^2z)\hat{k}$

at point $(1, 2, 1)$

$\nabla\phi = (12+2)\hat{i} - (3+1-16)\hat{j} + (2-32)\hat{k}$

$\nabla\phi = 14\hat{i} - 2\hat{j} - 30\hat{k}$

ii) $\nabla \cdot A = \frac{\partial a_x}{\partial x} + \frac{\partial a_y}{\partial y} + \frac{\partial a_z}{\partial z}$

$\nabla \cdot A = 2xy + (x+z) + 2xz$

at point $(1, 2, 1)$

$\nabla \cdot A = 4 + 2 + 2$

$\nabla \cdot A = 8$

iii) $\nabla \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ b_x & b_y & b_z \end{vmatrix}$

$\nabla \times B = \hat{i}\left(\frac{\partial b_z}{\partial y} - \frac{\partial b_y}{\partial z}\right) - \hat{j}\left(\frac{\partial b_z}{\partial x} - \frac{\partial b_x}{\partial z}\right) + \hat{k}\left(\frac{\partial b_y}{\partial x} - \frac{\partial b_x}{\partial y}\right)$

$\frac{\partial b_z}{\partial y} = 2x$, $\frac{\partial b_y}{\partial z} = -3x$, $\frac{\partial b_z}{\partial x} = 2y$

$\frac{\partial b_x}{\partial z} = y$, $\frac{\partial b_y}{\partial x} = -3z$, $\frac{\partial b_x}{\partial y} = z$

$\nabla \times B = (2x - (-3x))\hat{i} - (2y - y)\hat{j} + (-3z - z)\hat{k}$

$\nabla \times B = 5x\hat{i} - y\hat{j} - 4z\hat{k}$

at point $(1, 2, 1)$

$\nabla \times B = 5\hat{i} - 2\hat{j} - 4\hat{k}$

iv) grad div $A = \frac{\partial(\nabla\cdot A)}{\partial x}\hat{i} + \frac{\partial(\nabla\cdot A)}{\partial y}\hat{j} + \frac{\partial(\nabla\cdot A)}{\partial z}\hat{k}$

where $\nabla\cdot A = 2xy + (x+z) + 2xz$

$\bar{\nabla}(\bar{\nabla}\cdot A) = (2y+1+2z)\hat{i} + (2x)\hat{j} + (4+2x)\hat{k}$

at point $(1,2,1)$

$\bar{\nabla}(\bar{\nabla}\cdot A) = 7\hat{i} + 2\hat{j} + 3\hat{k}$

Curl curl $A = \bar{\nabla}\times(\bar{\nabla}\times A)$ let $\bar{\nabla}\times A = C$

$\nabla\times C = \left(\frac{\partial C_z}{\partial y} - \frac{\partial C_y}{\partial z}\right)\hat{i} - \left(\frac{\partial C_z}{\partial x} - \frac{\partial C_x}{\partial z}\right)\hat{j} + \left(\frac{\partial C_y}{\partial x} - \frac{\partial C_x}{\partial y}\right)\hat{k}$

$\bar{\nabla}\times A = \left(\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}\right)\hat{i} - \left(\frac{\partial a_z}{\partial x} - \frac{\partial a_x}{\partial z}\right)\hat{j} + \left(\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right)$

$A = x^2y\hat{i} + (xy+yz)\hat{j} + xz^2\hat{k}$

$\frac{\partial a_z}{\partial y} = 0$, $\frac{\partial a_y}{\partial z} = y$; $\frac{\partial a_z}{\partial x} = z^2$

$\frac{\partial a_x}{\partial z} = 0$, $\frac{\partial a_y}{\partial x} = y$, $\frac{\partial a_x}{\partial y} = x^2$

$\therefore \bar{\nabla}\times A = (0 - y)\hat{i} - (z^2 - 0)\hat{j} + (y - x^2)k$

$\bar{\nabla}\times A = -y\hat{i} - z^2\hat{j} + (y - x^2)k$

where $\bar{\nabla}\times A = C$

$\frac{\partial C_z}{\partial y} = 1$, $\frac{\partial C_y}{\partial z} = -2z$; $\frac{\partial C_z}{\partial x} = -2x$

$\frac{\partial C_x}{\partial z} = 0$, $\frac{\partial C_y}{\partial x} = 0$, $\frac{\partial C_x}{\partial y} = 1$

$\therefore \bar{\nabla}\times C = (1 - (-2z))\hat{i} - (-2x - 0)\hat{j} + (0 - (-1))k$

$\bar{\nabla}\times C = (1+2z)\hat{i} + 2x\hat{j} + \hat{k}$

at point $(1,2,1)$

$\bar{\nabla}\times C = \bar{\nabla}(\bar{\nabla}\times A) = (1+2)\hat{i} + 2\hat{j} + \hat{k}$

$\therefore \bar{\nabla}(\bar{\nabla}\times A) = 3\hat{i} + 2\hat{j} + \hat{k}$