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17/ENG01/034

CHEMICAL ENGINEERING

ENG282

Mathematical model is defined as a process of developing a mathematical model.

Methods of developing model in engineering system

1. Radioactivity.
2. Mixing problems.

Question 2.

1. $r = (t^2+3t)i - 2\sin 3t j + 3e^{2t}k$

a. $\frac{dr}{dt} = (2t+3)i - (6\cos 3t)j + (6e^{2t})k$

b. $\frac{d^2r}{dt^2} = (2)i + (18\sin 3t)j + (12e^{2t})k$

$\frac{d^2r}{dt^2}$ at $t=0$ $= 2i + 18\sin 0 j + 12e^0 k$

$\frac{d^2r}{dt^2}$ at $t=0$ $= 2i + 12k$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = \sqrt{2^2+12^2}$

$= \sqrt{4+144}$

$= \sqrt{148}$

$\left|\frac{d^2r}{dt^2}\right|_{t=0} = 12.2$ units

Question 3

$A = x^2yi + (xy+yz)j + xz^2k$

$B = yzi - 3xzj + 2xyk$

$\phi = 3x^2y + xyz + 4y^2z^2 - 3$

$$\nabla\phi = \left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \right]\phi$$

$$\nabla\phi = i\frac{\partial\phi}{\partial x} + j\frac{\partial\phi}{\partial y} + k\frac{\partial\phi}{\partial z}.$$

$\nabla\phi = i[yz + 6xy + j(3x^2 + xz - 8yz^2) + k(xy - 8y^2z)$

at point $(1, 2, 1)$

$\nabla\phi = 1(2\times1 + 12) + y(3\times1^2 + 1\times1 - 8\times2\times1^2) + k(1\times2 - 8\times2^2\times1)$

$\nabla\phi - 1(14) + j(-12) + k(-30)$

$\nabla\phi = 14 + 12j + 30k$

$\nabla\cdot A.$.

$$\nabla = i\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

$A = a_x i + a_y j + a_z k$

$$\nabla\cdot A = \left[\frac{i\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z} \right] \cdot (a_x i + a_y j + a_z k)$$

$$\nabla\cdot A = \frac{\partial}{\partial x}\cdot a_x + \frac{\partial}{\partial y}\cdot a_y + \frac{\partial}{\partial z}\cdot a_z$$

$= 2xy. + (x+2) + 2xz.$

$\nabla\cdot A$ at $(1,2,1)$

$\nabla\cdot A = 2\times1\times2 + (1+2)\times2\times1\times1$

$\nabla\cdot A = 4 + 3 + 2.$

$\nabla\cdot A = 9$

$$\nabla\times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -3xz & 2xy \end{vmatrix}$$

$$i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ -3xz & 2xy \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ yz & 2xy \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ y^2 & -3xz \end{vmatrix}$$

$i\left[\frac{\partial}{\partial y}(2xy) - \frac{\partial}{\partial z}(-3xz)\right] - j\left[\frac{\partial}{\partial x}(2xy) - \frac{\partial}{\partial z}yz\right] +$

$k\left[\frac{\partial}{\partial x}(-3xz) - \frac{\partial}{\partial y}y^2\right]$

$[2x + 3x)i - (2y - y)j + (-3z - 2)k$

$\nabla \times B$ at $(1,2,1)$

$\nabla \times B = (2\times1 + 3\times1)i - (2\times2 - 2)j - (-3\times1 - 2)k$

$\nabla \times B = 5i - 2j + 4k$

Grad of div A

$\nabla \cdot A = 2xy + (x+2) + 2xz$

$\nabla \cdot A = \left[i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right]\nabla \cdot A$

$= [(2y + 1 + 2z)i + (2x)j + k(1 + 2x)k]$

$\nabla(\nabla \cdot A)$ at $(1,2,1)$

$\nabla(\nabla \cdot A) = (2\times2 + 1 + 2\times1)i + (2\times1)j + (1 + 2\times1)k$

$= 7i + 2j + 3k$

$= 7i + 2j + 3k,$

Calculate A.

$\text{Curl } A = \nabla \times A \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & (xy+yz) & (xz^2) \end{vmatrix}$

$i\begin{vmatrix} \partial/\partial y & \partial/\partial z \\ (xy+yz) & (xz^2) \end{vmatrix} - j\begin{vmatrix} \partial/\partial x & \partial/\partial z \\ x^2y & (xz^2) \end{vmatrix} + k\begin{vmatrix} \partial/\partial x & \partial/\partial y \\ x^2y & (xy+y^2) \end{vmatrix}$

$i\left(\frac{\partial}{\partial y}(xz^2) - \frac{\partial}{\partial z}(xy+yz)\right) - j\left(\frac{\partial}{\partial x}(xz^2) - \frac{\partial}{\partial z}(x^2y)\right)$

$+\bar{k}\left[\frac{\partial}{\partial x}(xy+yz) - \frac{\partial}{\partial y}(x^2y)\right]$

$i[0 - y] - j(z^2 - 0) + k(y - x^2).$

$$\nabla \times A = -y\,i - z^2\,j + (y - x^2)\,k$$

$$\nabla \times (\nabla \times A) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y & -z^2 & (y - x^2) \end{vmatrix}$$

$$i\left[\frac{\partial}{\partial y}(y - x^2) - \frac{\partial}{\partial z}(-z^2)\right] - j\left[\frac{\partial}{\partial x}(y - x^2) - \frac{\partial}{\partial z}(-y)\right]$$

$$+ k\left[\frac{\partial}{\partial x}(-z^2) - \frac{\partial}{\partial y}(-y)\right]$$

$$i(1 + 2z) - j(-2x + 0) + k(0 + 1)$$

$\nabla \times (\nabla \times A)$ at $(1, 2, 1)$

$$\nabla \times (\nabla \times A) = i(1 \times 2 \times 1) - j(-2 \times 1) + k(1)$$

$$\nabla \times (\nabla \times A) = 3i + 2j + k.$$