NAME: OKAKURO BRUME
MAT NO: 16/ENG04/041
DEPT: ELECTRICAL/ELECTRONICS ENGINEERING
COURSE: ENGINEERING MATHEMATICS (ENG 282)
DATE: 7th of APRIL 2018

ASSIGNMENT

1i: Mathematical Modelling

This can be defined as a representation in mathematical terms of the behaviour of real devices, object or problems.

It can also be defined as the process of representing the behaviour of a real system by a collection of mathematical equations & logic

ii: Methods of obtaining model for Engineering Systems

Conservation and Balance Principles

Dimensional Homogeneity and Consistency

2 If $r = (t^2+3t)i - 2\sin 3t\, j + 3e^{2t}K$

i) $dr/dt = (2t+3)i - 6\cos 3t\, j + 6e^{2t}K$

ii) $d^2r/dt^2 = 2i + 18\sin 3t\, j + 12e^{2t}K$

iii) $\left|\dfrac{d^2r}{dt^2}\right|$ at $t=0$

$$\frac{d^2r}{dt^2} \text{ at } t=0 = 2i + 18\sin 3(0)j + 12e^{2(0)}K$$

$$\frac{d^2r}{dt^2} \text{ at } t=0 = 2i + 0j + 12K$$

$$\left|\frac{d^2r}{dt^2}\right|_{at\,t=0} = \sqrt{(2)^2 + (12)^2}$$

$$\left|\frac{d^2r}{dt^2}\right|_{at\,t=0} = 12.165 \text{ units}$$

(3) If

$A = x^2y\,i + (xy + yz)\,j + xz^2\,K$

$B = yz\,i - 3xz\,j + 2xy\,K$ and

$\phi = 3x^2y + xyz - 4y^2z^2 - 3$

determine, at the point $(1, 2, 1)$

(i) $\nabla\phi$ (ii) $\nabla\cdot A$ (iii) $\nabla\times B$ (iv) grad div A (v) Curl Curl A

Solution

(i) $$\nabla\phi = \frac{\partial\phi}{\partial x}i + \frac{\partial\phi}{\partial y}j + \frac{\partial\phi}{\partial z}K$$

$$\nabla\phi = \frac{\partial(3x^2y + xyz - 4y^2z^2 - 3)}{\partial x}i + \frac{\partial(3x^2y + xyz - 4y^2z^2 - 3)}{\partial y}j + \frac{\partial(3x^2y + xyz - 4y^2z^2 - 3)}{\partial z}K$$

$$\nabla\phi = [6xy + yz]\,i + [3x^2 + xz - 8yz^2]\,j + [xy - 8y^2z]\,K$$

$x = 1,\ y = 2,\ z = 1$

$$\nabla\phi = [6(1)(2) + (2)(1)]\,i + [3(1)^2 + (1)(1) - 8(2)(1)^2]\,j + [(1)(2) - 8(2)^2(1)]\,K$$

$$\nabla\phi = 14i - 12j - 30K$$

(ii) $$\nabla\cdot A = \left[\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}K\right]\cdot\left[a_x i + a_y j + a_z K\right]$$

$$\nabla\cdot A = \left[\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}K\right]\cdot\left[x^2y\,i + (xy + yz)\,j + xz^2\,K\right]$$

$i\cdot i = j\cdot j = K\cdot K = 1$

$$\nabla\cdot A = \frac{\partial(x^2y)}{\partial x} + \frac{\partial(xy + yz)}{\partial y} + \frac{\partial(xz^2)}{\partial z}$$

$\nabla\cdot A = 2xy + (x+2) + 2xz$

$x=1,\ y=2,\ z=1$

$\nabla\cdot A = 2(1)(2) + (1+1) + 2(1)(1)$

$\nabla\cdot A = 8$

(iii) $\nabla\times B = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ b_x & b_y & b_z \end{vmatrix} = i\left(\frac{\partial b_z}{\partial y} - \frac{\partial b_y}{\partial z}\right) - j\left(\frac{\partial b_z}{\partial x} - \frac{\partial b_x}{\partial z}\right)$

$+ k\left(\frac{\partial b_y}{\partial x} - \frac{\partial b_x}{\partial y}\right)$

$\nabla\times B = i\left[\frac{\partial(2xy)}{\partial y} - \frac{\partial(-3xz)}{\partial z}\right] - j\left[\frac{\partial(2xy)}{\partial x} - \frac{\partial(yz)}{\partial z}\right] + k\left[\frac{\partial(-3xz)}{\partial x} - \frac{\partial(yz)}{\partial y}\right]$

$\nabla\times B = i(2x+3x) - j(2y-y) + k(-3z-z)$

$x=1,\ y=2,\ z=1$

$\nabla\times B = i(2(1)+3(1)) - j(2(2)-2) + k(-3-1)$

$\nabla\times B = 5i - 2j - 4k$

(iv) grad div $A = \nabla\cdot\nabla\cdot A$

$\nabla\cdot\nabla\cdot A = \nabla\cdot[2xy + (x+z) + 2xz]$

$\nabla\cdot\nabla\cdot A = \frac{\partial i}{\partial x} + \frac{\partial j}{\partial y} + \frac{\partial k}{\partial z}\cdot[2xy + (x+z) + 2xz]$

$\nabla\cdot\nabla\cdot A = 2y \cancel{+ 0} \overset{(1+z)}{+} \cancel{2z} + 2x \cancel{+ (1+2x)}$ $\quad (2y+1+2z)i + j(2x) + k$

$2y + (1+0) + 2z + 2x + (1+2x)$ $\quad (1+2x)$

$\cancel{\nabla\cdot A =}$ $2y+1+2z+2x+1+2x$ $\quad x=1,\ y=2,\ z=1$

$\nabla\cdot\nabla\cdot A = \cancel{\quad}$ $\quad x=1,\ y=2,\ z=1$ $\quad [2(2)+1+2]i + j(2) + [1+2]k$

$\nabla\cdot\nabla\cdot A = 2(2)+1+2(1)+2(1)+1+2(1)$

$\nabla\cdot\nabla\cdot A = 12$

$\nabla\cdot\nabla\cdot A = 7i + 2j + 3k$

grad div $A = 7i + 2j + 3k$

Curl Curl A = $\nabla \times \nabla \times A$

$$\nabla \times A = i\left(\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}\right) - j\left(\frac{\partial a_z}{\partial x} - \frac{\partial a_x}{\partial z}\right) + k\left(\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right)$$

$$\nabla \times A = i\left[\frac{\partial xz^2}{\partial y} - \frac{\partial(xy+yz)}{\partial z}\right] - j\left(\frac{\partial xz^2}{\partial x} - \frac{\partial x^2y}{\partial z}\right) + k\left(\frac{\partial(xy+yz)}{\partial x} - \frac{\partial x^2y}{\partial y}\right)$$

$$\nabla \times A = i[0 - y] - j[z^2 - 0] + k[y - x^2]$$

$$\nabla \times A = -yi - z^2j + (y - x^2)k$$

$$\nabla \times \nabla \times A = i\left[\frac{\partial(y-x^2)}{\partial y} - \frac{\partial(-z^2)}{\partial z}\right] - j\left[\frac{\partial(y-x^2)}{\partial x} - \frac{\partial(-y)}{\partial z}\right] + k\left[\frac{\partial(-z^2)}{\partial x} - \frac{\partial(-y)}{\partial y}\right]$$

$$\nabla \times \nabla \times A = i[1 + 2z] - j[-2x - 0] + k[0 + 1]$$

$$\nabla \times \nabla \times A = i[1 + 2z] - j[-2x] + k[1]$$

when $x = 1,\ y = 2,\ z = 1$

$$\nabla \times \nabla \times A = i[1 + 2] - j[-2] + k$$

$$\nabla \times \nabla \times A = 3i + 2j + k$$

Curl Curl A = $3i + 2j + k$